Ms. Nguyen Calculus 101 A
Sample Test Chapter 1

1. Let $f(x) = \dfrac{e^x - 1}{x}$.

   (a) Evaluate $f(x)$ for x close to zero and then estimate $\lim\limits_{x \to 0} \dfrac{e^x - 1}{x}$

   (b) Use the result from part (a) to evaluate the limit $\lim\limits_{x \to 0} \dfrac{2e^{2x} - 2}{x}$

2. Let $f(x) = \begin{cases} 3x - 1, & \text{if } x < 1 \\ -4 + 2x^2, & \text{if } x > 1 \end{cases}$. Find $\lim\limits_{x \to 1} f(x)$

3. Determine whether the statement is true or false. If it is false, give an example to show that it is false.

   If $\lim\limits_{x \to 3} f(x) = -2$, then $f(3) = -2$.

4. Find the limit: $\lim\limits_{x \to -1} \dfrac{x^2 + 3x + 2}{x^2 + 1}$

5. Find the limit: $\lim\limits_{x \to 1} \dfrac{2x - 3 + x^2}{(x - 1)(x + 1)}$

6. Find the limit: $\lim\limits_{x \to 0} \dfrac{x + x\cos x}{\sin x \cos x}$

7. Find the limit: $\lim\limits_{x \to 0} \dfrac{\tan 5x}{\tan 7x}$

8. Find the limit: $\lim\limits_{x \to \infty} \tan^{-1} 2x$

9. Find the limit: $\lim\limits_{x \to -\infty} \left(\sqrt{3x^2 + 5x + 2} - \sqrt{3x^2 - 2x + 5} \right)$.
   (Hint: multiply and divide by the conjugate)

10. Find the slant asymptote and vertical asymptote for
    $f(x) = \dfrac{2x^4 + 3x^3 - 2x - 4}{x^3 - 1}$.

11. The gravitational force exerted by the earth on an object having mass m that is a distance r from the center of the earth is

$$g(r) = \begin{cases} \dfrac{GMm}{r^2}, & \text{if } r \geq R; \\ \dfrac{GMmr}{R^3}, & \text{if } r < R. \end{cases}$$

    where G is the gravitational constant, M is the mass of the earth, and R is the earth's radius. Is $g(r)$ a continuous function of r? Justify your answer.

12. Find the x-values (if any) at which $f(x) = \dfrac{x^2 - 6x + 5}{x - 6}$ discontinuous. Is this discontinuity removable? If yes, define a new function that removes the discontinuity.

13. Prove that if $f(x)$ is continuous on $[0, 1]$ and satisfies $0 \leq f(x) \leq 1$, then there is a number $c \in [0, 1]$ such that $f(c) = c$.

14. Use the Intermediate Value Theorem to show that the function $f(x) = 2x^2 - 4$ has a zero in the interval $[1, 2]$. Use the method of bisections to find an interval of length 1/32 that contains the zero.

15. Find $\lim\limits_{x \to 2^-} \dfrac{x - 4}{x^2 - 4x + 4}$