

Mr. Bradshaw  
Math 101A Ch. 4 Sample Test

1. Find the antiderivatives of the following.

(a)  $\int \sin\left(\frac{x}{2}\right) dx$

(c)  $\int \sqrt{2x-1} dx$

(b)  $\int \frac{\sin 3x}{\cos 3x} dx$

(d)  $\int \frac{x^2 + 5x + 1}{x} dx$

2. Find the average value of  $y = \sin^2 3x$  over the interval  $x \in [0, \pi]$ .

Hint:  $\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$ .

3. Use an infinite sum of left endpoint rectangles to find the area under  $y = 2x^2 + 3$  in the interval  $x \in [0, 4]$ . Hint:  $\sum_{i=1}^{\infty} i^2 = \frac{n(n+1)(2n+1)}{6}$ .

4. Use 4 midpoint rectangles to estimate the area under  $y = 2 + \cos \pi x$  on the interval  $x \in [0, 1]$ .

5. The distance across a pond is measured every 10 feet. The values are given in the table. Use Simpson's Method to estimate the area of the pond.

$$\int_a^b f(x) dx \approx \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

$x$	0	10	20	30	40	50	60
distance	45	53	59	70	48	35	10

6. Use your calculator to estimate the area under  $y = e^{-x^3}$  in the interval  $x \in [0, 0.1]$

7. How many intervals must be used with Simpson's Method to find the area under  $y = \cos x$  on the interval  $x \in [0, 1]$  so that the error is

less than 0.00001?  $E = \frac{\max|f^{(4)}(x)|(b-a)^5}{180n^5}$