Math 101B     Test #3     Spring 2007     Name________________________

All 9 problems are worth 11 points each. That is: 4 points for setting the problem up correctly, 4 points for doing the calculus correctly, and the last 3 points for finding each exact answer (like All 9 problems are worth 11 points each. Remember: For problems where the answer is in the form of the series "diverges" or "converges", only two points for the answer "diverges" or "converges" and the rest of the points for using the correct test.

1. Sequences: Use L'Hopital's Rule *c*orrectly to determine if the following sequence converges or diverges: $a_n = \dfrac{n2^n}{3^n}$.

2. Infinite Series: Determine if the following series converges or diverges. If it converges, find the sum of the series.

$$\sum_{k=1}^{\infty} \frac{9}{k(k+3)}$$

3. Geometric Series: A hard rubber ball is dropped from a height of 16 feet onto a concrete floor. The ball always rebounds three-fourths of the distance fallen. Calculate the total distance the ball travels.

4. Use the Integral Test to determine if the following series converges or diverges. (Make sure you show that the $f(x)$ you choose is continuous, decreasing and non-negative over the appropriate interval that represents the series.)

$$\sum_{k=2}^{\infty} \frac{3}{k(\ln k)^2}$$

5. Alternating Series: Determine the number of terms needed to estimate the sum of the following series within $10^{-6}$.

$$\sum_{k=1}^{\infty} (-1)^k \frac{3}{k^2}$$

6. Determine whether the following series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt{k}}$$

7. Power Series: Find a power series to represent the following function and find the radius of convergence.

$$f(x) = \frac{3}{3+x^2}$$

8. Taylor Series: Write the first four terms of the Taylor Series for the following function:

$$f(x) = \frac{1}{\sqrt{x}} \text{ centered around } x = 1.$$

9. Applications of Taylor Series: Use the first five terms of this series $e^x = \displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k!}$

to estimate $\displaystyle\int_0^2 e^{-3x^2}\,dx$