Math 101C Ch. 13 SampleTest (from Spring 2009)

1. Sketch the region in the first quadrant bounded by $x = 0$, $y = \frac{3}{4}x$, and $y = \sqrt{100 - x^2}$.

   (a) Set up the double integral $\iint dydx$ that represents this area.

   (b) Set up the double integral $\iint dxdy$ that represents this area.

   (c) Set up the double integral in polar coordinates that represents this area.

2. Sketch the region described by $\int_0^1 \int_2^{4-2x} dy\, dx$.

3. Sketch the region described by $\int_0^{\pi} \int_2^4 r\, dr\, d\theta$.

4. Write the integral described by $\int_{\pi/4}^{3\pi/4} \int_0^{4 \csc\theta} r\, dr\, d\theta$ as a double integral in rectangular coordinates.

5. Set up but do not solve the double integral in polar coordinates that gives the volume between the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.

6. Set up but do not solve the integral in spherical coordinates that gives the volume between the paraboloid $z = 2(x^2 + y^2)$ and the plane $z = 4$.

7. Set up but do not solve the integral that gives the mass under the paraboloid $z = 9 - x^2 - y^2$, with $z \geq 0$ if the density of the object is given by $\delta(x, y, z) = x + y + 2z$.

8. Set up but do not solve the integral in polar coordinates that gives the surface area of the paraboloid $z = 9 - x^2 - y^2$, with $z \geq 0$.

9. Set up but do not solve the integral in rectangular coordinates that gives the surface area of $x = \cos u, y = \sin u \sin v, z = \cos u \cos v$, with $0 \leq u \leq \frac{\pi}{4}$ and $0 \leq v \leq \frac{\pi}{4}$.

10. Set up and solve the integral in polar coordinates that gives the volume inside both the sphere $x^2 + y^2 + z^2 = a^2$ and the cylinder $x^2 - ax + y^2 = 0$. You may want to use $\cos^4 t = \frac{1}{8}(3 + 4\cos 2t + \cos 4t)$