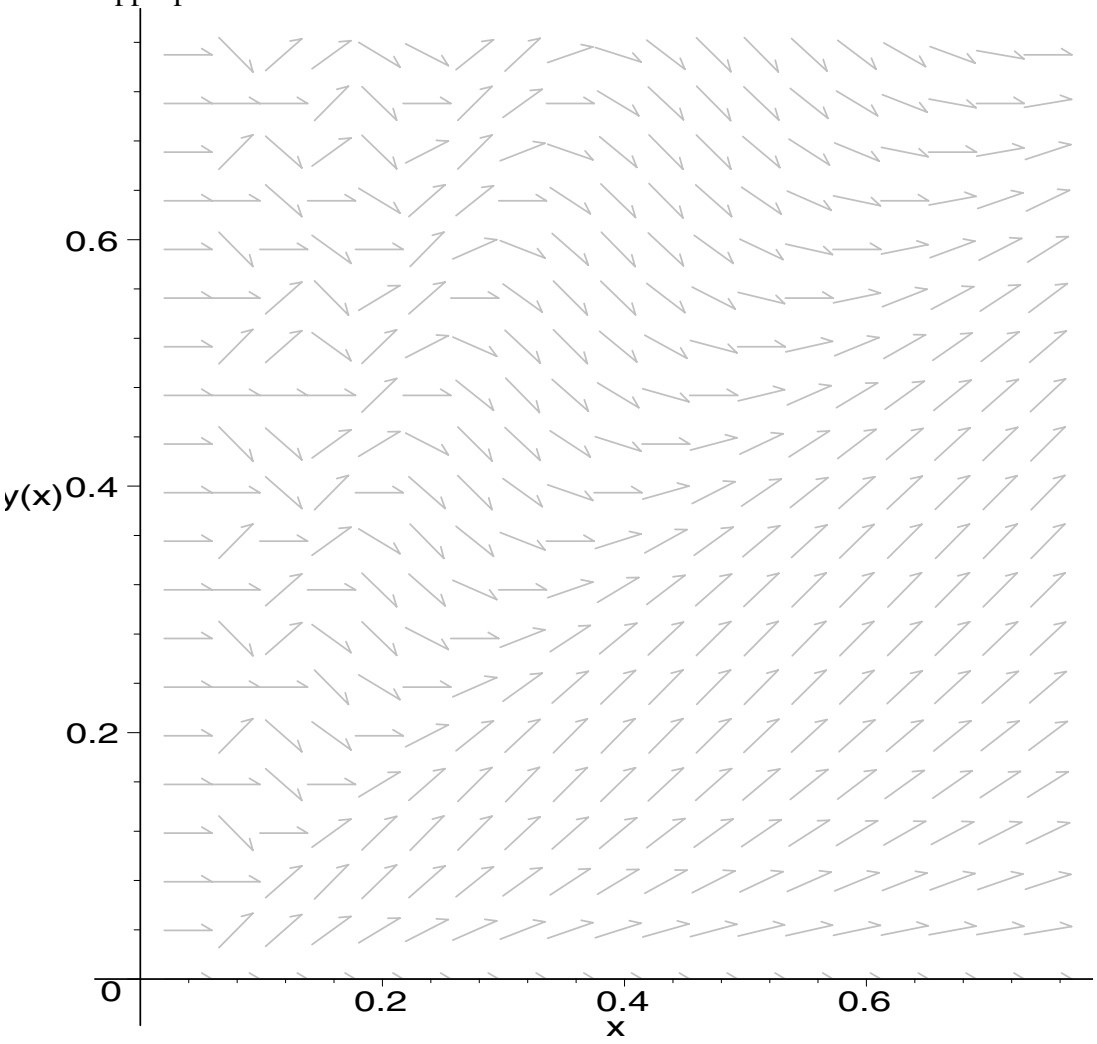
Math 104 Ch 1 and 2 Test Name _____

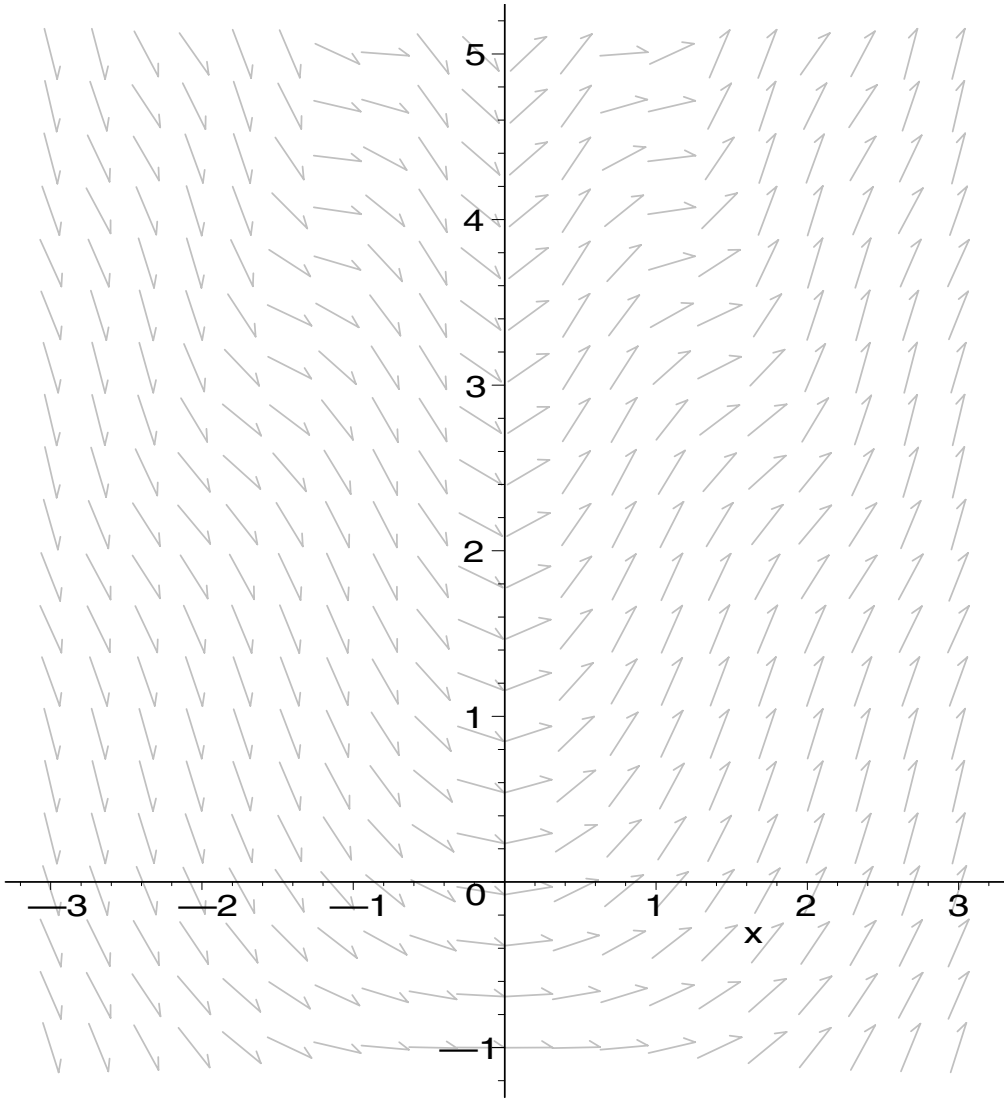
Show all you work if you want to receive credit.

(1) Given the differential equation $\frac{dy}{dx} = \sin\left(\frac{\pi y}{x}\right)$ and the slope field shown below, draw three isoclines when $\frac{dy}{dx}$ is equal to 1. Label the isoclines with the appropriate values.



(2) Does the above equation have an isocline value of –1.2? Explain why or why not.

(3) Given the slope field below, draw the solution curves through (–1, 3) and (1, 0).



(4) Given the equation $\frac{dx}{dt} = x(10-x)(20-x)^2$

(a) Draw the phase diagram and indicate stability types

(b) Draw several solution curves above and below each stability level

(5) In a cascade of two tanks, the first tank initially contains 100 gallons of pure ethanol and the second tank initially contains 100 gallons of pure water. Pure water flows into the first tank at 10 gal/min. The well mixed solution from the first tank flows into the second tank at 10 gal/min. The well mixed solution leaves the second tank at 10 gal/min and is discarded.

(a) Find the amount of ethanol in the first tank at any time t.

(b) Find the amount of ethanol in the second tank at any time t.

(6) A population $P(t)$ of rodents has birth rate $\beta = 0.001P$ (births per month per rodent) and a constant death rate of zero. If $P(0) = 100$,how long (in months) will it take this population to double to 400 rodents?

(7) Consider a savings account that contains \$5000 dollars initially and earns interest at the annual rate 10% compounded continuously. Suppose that interest is allowed to accrue and that additional deposits are added to this account at the rate of Q dollars per year, also at a continuous rate. You wish to have \$100,000 in the account in 18 years. What value of Q will allow you to reach your goal?

(8) Suppose that an initially full hemispherical tank of radius 3 ft has its flat side as its bottom. A hole in the bottom of the tank is a square of side 1/2 in. If this bottom hole is to be opened at 1 PM, when will the tank be empty?

Use Torricelli's Law: $\frac{dV}{dt} = -a\sqrt{2gy}$ where g = 32 ft/sec^2, V = volume, y = height of water, a = area of the hole.