

Show all your work if you want to receive credit.

(1) Find the convolution of $f(t) = e^t$ and $g(t) = t$.

(2) Find the transforms of the following ($\sin(A + B) = \sin A \cos B + \sin B \cos A$)

(a) $t \sin 3t$

(b) $f(t) = \begin{cases} \cos \frac{\pi t}{2} & 3 \leq t \leq 6 \\ 0 & \text{otherwise} \end{cases}$

(3) Find the inverse transforms of

(a) $\frac{s(1 - e^{-2s})}{s^2 + \pi^2}$

(b) $\frac{1}{as^2} - \frac{e^{-as}}{s(1 - e^{-as})}$

(4) Solve the following using Laplace transforms

(a) $x'' + 4x = 2, x(0) = 1, x'(0) = 0$

(b) $tx'' + (t - 2)x' + x = 0, x(0) = 0$

(c) $x'' + 2x' + x = \delta(t) - \delta(t - 2), x(0) = x'(0) = 0$

(d) $x'' + x = \begin{cases} t & 0 \leq t \leq 1 \\ 0 & t > 1 \end{cases}, x(0) = 0, x'(0) = 0$

Use $\frac{s}{s(s^2 + 1)} = \frac{1}{s} - \frac{s}{s^2 + 1}$.

Show all the details in figuring out the other partial fractions