Exam 2 Last Name____________________________
Math 163/CS113 First Name____________________________

Directions: Use the space assigned to solve problems. Read each problem carefully. Show all work for any credit.

1. Prove the statement by the method of exhaustion: Every positive even integer n, such that $4 \le n \le 30$ can be expressed as a sum of two prime numbers.

2. Disprove the statement by giving a counterexample. For all real numbers a , $a^2 - 1 > 0$.

Prove the statements that are true and give counterexamples to disprove those that are false. (#3 & #4)

3. Any product of four consecutive integers is one less than a perfect square.

4. The difference of any two odd integers is odd.

5. Suppose m is an integer such that $2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 9 \cdot m = 17 \cdot 24 \cdot 30 \cdot 63 \cdot 144$. Does 17 divide m? Why?

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6. Write the number as a ratio of two integers: 12.73131313…

Give a reason for your answer in each of the exercises. Assume that all variables represent integers. (#7 & #8)

7. Does $9 \mid 78$?

8. If $n = 4k + 1$, does $8 \mid n^2 - 1$?

9. Prove the statement. For all integers a, b, c, if $a \mid b$ and $a \mid c$ then $a \mid (b + c)$

10. Compute $\lfloor x \rfloor$ and $\lceil x \rceil$ for each of the values of x:

a) $\frac{17}{8}$

b) -67 / 7

11. Use the floor notation to express 685 div 14 and 685 mod 14.

12. Carefully formulate the negations of the statement. Then prove the statement by contradiction. There is no greatest even integer.

13. By experimenting with small values of n, guess a formula for the given sum then prove it by mathematical induction: $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}=$

14. The statement is either true or false. Prove if it is true and disprove if it is false. $2 - \sqrt{5}$ is irrational.

15. Write the first four terms of the sequence defined by the formulas. $b_j = (-1)^j + 2^j$, for all integers $j \geq 0$.

16. Find the explicit formula for the sequence with the initial terms given. 0, 1, -2, 3, -4, 5…

17. Write using summation or product notation. $(2^2 + 1)(3^2 + 1)(4^2 +1)(5^2 +1)(6^2 +1)$.

18. Transform each by making the change of variable j = i + 1.

a) $\sum_{i=0}^{n-1}(i^2+2i-1)$ b) $\prod_{i=1}^{n}(i-2)^2$

19. Prove by contraposition: If sum of two real numbers is greater than 100, then at least one of the numbers is greater than 50.

20. Prove the statement by mathematical induction: $11^n - 6$ divisible by 5, for each integer $n \geq 1$.