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OHLONE COLLEGE

Ohlone College Placement Center Arithmetic Study Guide

Developed by Aims Community College

Fractions

Terms

Fraction: A type of number that represents division.
Examples: $\frac{1}{3} = 1 \div 3$, $\frac{19}{5} = 19 \div 5$.

Numerator: The number on the top of the fraction.
Example: The numerator of $\frac{3}{4}$ is 3.

Denominator: The number on the bottom of the fraction.
Example: The denominator of $\frac{3}{4}$ is 4.

Proper fraction: The top number is less than the bottom number.
Examples: $\frac{1}{3}$, $\frac{7}{10}$, $\frac{9}{19}$

Improper fraction: The top number is equal to or larger than the bottom number.
Examples: $\frac{3}{2}$, $\frac{9}{4}$, $\frac{8}{8}$

Mixed number: A whole number is written next to a proper fraction.
Examples: $1\frac{3}{4}$, $2\frac{2}{5}$

Common denominator: A number that can be divided evenly by the denominators of several fractions.
Example: The fractions $\frac{1}{4}$ and $\frac{5}{6}$ have a common denominator of 12 since both 4 and 6 divide evenly into 12.

Reducing Fractions to Lowest Terms

Since a fraction is a number that represents division, fractions such as $\frac{4}{2}$ and $\frac{6}{3}$ are equivalent since both fractions have a value of 2. Similarly, fractions such $\frac{2}{4}$ and $\frac{1}{2}$ have the same value. One common operation done with fractions is to **reduce the fraction to lowest term**, meaning to write that fraction using numbers with values as low as possible. This is accomplished by dividing both the numerator and denominator by some common factor.

Example: Write $\frac{48}{64}$ in lowest terms.

$\frac{48}{64} = \frac{48 \div 8}{64 \div 8} = \frac{6}{8}$	Find a number that divides evenly into both the numerator and denominator of the fraction. For the fraction on the left, there are several choices, including 2, 4, and 8. We start by dividing the top and bottom of the fraction by 8.
$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$	Check to see if the fraction can again be reduced. If so, reduce the fraction again. If not, the fraction is in lowest terms. In the example at the left, division by 2 reduces the fraction to lowest terms.

Changing a Mixed Number to an Improper Fraction

To change a mixed number into an improper fraction,

1. Multiply the whole number part by the denominator of the fraction.
2. Add this result to the original numerator.
3. Write the fraction using this result as the numerator and the original denominator.

Example: Change $2\frac{3}{4}$ to an improper fraction.

$2 \times 4 = 8$	Multiply the whole number by the denominator.
$8 + 3 = 11$	Add this result to the original numerator.
$\frac{11}{4}$	Write the fraction using this result as the numerator and the original denominator.

Changing an Improper Fraction to a Mixed Number

To change a mixed number into an improper fraction,

1. Divide the numerator of the fraction by the denominator.
2. Use the result of the division as the whole number part of the mixed number.
3. Use the remainder from the division as the numerator of the fractional part of the mixed number.
4. If necessary, reduce the fraction part of the mixed number to lowest terms.

Example: Change $\frac{26}{6}$ to a mixed number.

$\frac{26}{6} = 28 \div 6 = 4$ with remainder 2.	Divide the numerator of the fraction by the denominator.
$\frac{26}{6} = 4\frac{2}{6}$	Use the write the result of the division as a mixed number.
$4\frac{2}{6} = 4\frac{1}{3}$	Reduce the fractional part o the mixed number.

Multiplying and Dividing Fractions

To multiply fractions,

1. Multiply the numerators together and multiply the denominators together.
2. Check that the result is in lowest terms and, if necessary, as a mixed number.

Example

$\frac{3}{4} \times \frac{5}{6} = \frac{15}{24}$	Start by multiplying the numerators together and multiplying the denominators together.
$\frac{15}{24} = \frac{15 \div 3}{24 \div 3} = \frac{5}{8}$	Since both the numerator and denominator are divisible by 3, reduce the fraction.

To divide fractions,

1. Invert (flip over) the divisor (the second fraction) and multiply the resulting fractions.
2. Check that the result is in lowest terms and, if necessary, as a mixed number.

Example

$\frac{8}{3} \div \frac{5}{6} = \frac{8}{3} \times \frac{6}{5} = \frac{48}{15}$	Start by inverting the second fraction and multiply the fractions together.
$\frac{48}{15} = \frac{48 \div 3}{15 \div 3} = \frac{16}{5} = 3\frac{1}{5}$	Since both the numerator and denominator are divisible by 3, reduce the fraction.

To multiply mixed numbers,

1. Convert the mixed numbers into improper fractions.
2. Multiply the resulting fractions.
3. Write the result as a mixed number in lowest terms.

Example:

$4\frac{1}{3} \times 1\frac{1}{2} = \frac{13}{3} \times \frac{3}{2}$	Convert the mixed numbers into improper fractions.
$\frac{13}{3} \times \frac{3}{2} = \frac{39}{6}$	Perform the multiplication.
$\frac{39 \div 3}{6 \div 3} = \frac{13}{2} = 6\frac{1}{2}$	Convert this answer to a mixed number.

Adding and Subtracting Fractions

To add or subtract fractions,

1. Write the fractions so that they have a common denominator.
2. Combine the numerators.
3. Check that the result is in lowest terms and, if necessary, as a mixed number.

Example 1:

$\frac{1}{5} + \frac{2}{5}$	Since the denominators are the same, add the numerators.
$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$	Since this result is in lowest terms, this is the final answer.

Example 2:

$\frac{3}{4} + \frac{2}{3}$	Since the denominators are the different, convert the fractions so that they have a common denominator.
$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$ $\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$	Since the common denominator for the two fractions is 12, multiply the first fraction by $\frac{3}{3}$ and the second fraction by $\frac{4}{4}$.
$\frac{9}{12} + \frac{8}{12} = \frac{17}{12}$	Add the fractions and then convert this answer to a mixed number.
$\frac{17}{12} = 1\frac{5}{12}$	

Example 3:

$4\frac{1}{3} - 3\frac{2}{5} = \frac{13}{3} - \frac{17}{5}$	Convert the mixed numbers into improper fractions.
$\frac{13}{3} \times \frac{5}{5} = \frac{65}{15}$ $\frac{17}{5} \times \frac{3}{3} = \frac{51}{15}$	Write the fractions with a common denominator.
$\frac{65}{15} - \frac{51}{15} = \frac{14}{15}$	Subtract the fractions.

Practice Problems for Fractions

1. Change $4\frac{1}{6}$ into an improper fraction.
2. Change $\frac{42}{16}$ to a mixed number.
3. $5\frac{3}{5} + 2\frac{2}{3}$
4. $5\frac{1}{2} + 3\frac{2}{3}$
5. $9\frac{11}{13} - 2\frac{1}{2}$
6. $4\frac{1}{8} - 2\frac{3}{4}$
7. $3\frac{1}{7} \times \frac{5}{9}$
8. $3\frac{3}{7} \times 2\frac{7}{9}$
9. $\frac{6}{11} \div 14$
10. $3\frac{4}{5} \div 5\frac{5}{6}$

Answers to Practice Problems

1. $\frac{25}{6}$
2. $2\frac{5}{8}$
3. $8\frac{4}{15}$
4. $9\frac{1}{6}$
5. $7\frac{9}{26}$
6. $1\frac{3}{8}$
7. $1\frac{47}{63}$
8. $9\frac{11}{21}$
9. $\frac{3}{77}$
10. $\frac{114}{175}$

Decimals

Adding and Subtracting Decimals

To add or subtract decimals,

1. Arrange the numbers in a column, aligned at the decimal point.
2. Add (or subtract) the values.

Example: $28.5 + 44.47 + 3000.0003$

$$\begin{array}{r} 28.5 \\ 44.47 \\ +\ 3000.0003 \\ \hline 3072.9703 \end{array}$$

Multiplying Decimals

To multiply decimals,

1. Multiply the numbers as if they were whole numbers.
2. Count the total number of digits to the right of the decimal point in both of the original numbers.
3. Place the decimal point so that the final answer has this number of digits to the right of the decimal point.

Example: 18.9×5.03

$$\begin{array}{r} 189 \\ \times 503 \\ \hline 567 \\ 94500 \\ \hline 95\overrightarrow{067} \end{array}$$

Since the original numbers, 18.9 and 5.03 have a total of three places to the right of the decimal point, the final answer must have a total of three places to the right of the decimal point. Therefore, the final answer is 95.067.

Dividing a Decimal by a Whole Number.

Example: $2.701 \div 73$

$$\begin{array}{r} 0.037 \\ 73\overline{)\,2.701} \\ 2\ 19 \\ \hline 511 \\ 511 \\ \hline 0 \end{array}$$

Place the decimal point directly above its position inside the division sign. Divide the numbers if they were whole numbers.

Dividing Two Decimal Numbers

Example: $0.39603 \div 0.0043$

$0.0043\overline{)0.39603} \rightarrow 43\overline{)3960.3}$

Move the decimal point of the divisor (outside the bracket) to the far right. Move the decimal point of the dividend (inside the bracket) the same number of places to the right.

$$\begin{array}{r} 92.1 \\ 43\overline{)\,3960.3} \\ \underline{387} \\ 90 \\ \underline{86} \\ 43 \\ \underline{43} \\ 0 \end{array}$$

Place the decimal point directly above its position inside the division sign. Divide the numbers if they were whole numbers.

Practice Problems for Decimals

1. 18.1×0.04
2. 0.97×5.6
3. $123 + 2.6 + 9.04$
4. $83.0097 + 124.9 + 9.043$
5. $0.07 - 0.002$
6. $96 - 0.3992$
7. $27.36 \div 4$
8. $0.2601 \div 9$
9. $7.055 \div 0.83$
10. $4.466 \div 2.03$

Answers to Practice Problems

1. 0.724
2. 5.432
3. 134.64
4. 216.9527
5. 0.068
6. 95.6008
7. 6.84
8. 0.0289
9. 8.5
10. 2.2

Percents

Percents are used to describe a part of something. For example, percents are used to find the sales tax on the purchase of an item. When converting a percent to its fraction form, the fraction will always have a denominator of 100.

Changing Decimals to Percents

When changing a decimal to a percent, move the decimal point two places to the right and attach the percent sign.

Example: $0.35 = 0.\underset{\longrightarrow}{35} = 35\%$

$0.8 = 0.\underset{\longrightarrow}{80} = 80\%$

Changing Percents to Decimals

When changing a percent to a decimal, remove the percent sign and move the decimal point two places to the left.

Example $30\% = \underset{\longleftarrow}{30} = 0.30$

$0.9\% = \underset{\longleftarrow}{0.}9 = 0.009$

Changing Fractions to Percent Form

Divide the bottom number of the fraction into the top number and move the decimal point two places to the right.

Example:
$$\begin{array}{r} 0.75 \\ 4\overline{)\,3.00} \\ \underline{2\,8} \\ 20 \\ \underline{20} \\ 0 \end{array} \qquad 0.75 = 75\%$$

Changing Percents to Fractions

To change percents to fractions, write the percentage as a fraction, using 100 as the denominator. Then, reduce the fraction.

Example: $85\% = \dfrac{85}{100} = \dfrac{85 \div 5}{100 \div 5} = \dfrac{17}{20}$

Percent of a Number

To find a percent of a number, change the percent to a decimal and multiply the decimal by the number.

Example: What is 25% of \$6500?

$n = 25\% \times \$6500$

$n = 0.25 \times \$6500$

$n = \$1625$

Finding What Percent of One Number is Another Number

To determine what percent of one number is another number, use a proportion.

Example: What percent of 45 is 9?

$$\frac{p}{100} = \frac{9}{45}$$

$$p = \frac{9}{45} \times 100$$

$$p = 20 \Rightarrow p = 20\%$$

Finding a Number When a Percent of It is Given

Example: 24% of what number is 18?

$$\frac{24}{100} = \frac{18}{n}$$

$$24n = 18 \times 100$$

$$n = \frac{1800}{24}$$

$$n = 75$$

Practice Problems for Decimals

Write the following in percent form.

1. 0.12
2. $\frac{6}{8}$
3. 0.233
4. 1.15

Answer the following.

5. Write 125% as a decimal
6. What is 11% of \$3000?
7. 28 is 40% of what number?
8. 60 is what percent of 12,000?

Answers to Practice Problems

1. 12%
2. 75%
3. 23.3%
4. 115%
5. 1.25
6. \$330
7. 70
8. 0.5%