

Ohlone College Placement Center College Level Math Study Guide

Developed by Aims Community College

I Factor the following polynomials.

- $15a^3b^2 - 45a^2b^3 - 60a^2b$
- $7x^3y^3 + 21x^2y^2 - 10x^3y^2 - 30x^2y$
- $6x^4y^4 - 6x^3y^2 + 8xy^2 - 8$
- $2x^2 - 7xy + 6y^2$
- $y^4 + y^2 - 6$
- $7x^3 + 56y^3$
- $81r^4 - 16s^4$
- $(x + y)^2 + 2(x + y) + 1$

II Expand the following polynomials.

- $(x + 1)(x - 1)(x - 3)$
- $(2x + 3y)^2$
- $(\sqrt{3}x + \sqrt{3})(\sqrt{6}x - \sqrt{6})$
- $(x^2 - 2x + 3)^2$
- $(x + 1)^5$
- $(x - 1)^6$

III Simplifying Rational Expressions

Simplify the following. Assume all variable are greater than zero.

- $3^2 + 5 - \sqrt{4} + 4^0$
- $9 \div 3 \cdot 5 - 8 \div 2 + 27$

3. $\sqrt{\frac{81}{x^4}}$

4. $2\sqrt{18} - 5\sqrt{32} + 7\sqrt{162}$

IV Solve the linear equations.

- $3 - 2(x - 1) = x - 10$
- $\frac{x}{2} - \frac{x}{7} = 1$
- $y(y + 2) = y^2 - 6$
- $2[x - (1 - 3x)] = 3(x + 1)$

V Solve the polynomial equations.

- $\left(y - \frac{8}{3}\right)\left(y + \frac{2}{3}\right) = 0$
- $2x^3 - 4x^2 - 30x = 0$
- $27x^3 = 1$
- $(x - 3)(x + 6) = 9x + 22$
- $t^2 + t + 1 = 0$
- $3x^3 = 24$
- $(x + 1)^2 + x^2 = 25$
- $5y^2 - y = 1$

VI Solve the rational equations.

- $\frac{1}{y - 1} + \frac{2}{y + 1} = 0$
- $\frac{2}{x - 3} - \frac{3}{x + 3} = \frac{12}{x^2 - 9}$

$$3. \frac{1}{6-x} + \frac{2}{x+3} = \frac{5x}{x^2-3x-18}$$

$$4. \frac{11}{x^2-25} - \frac{2}{x-5} = \frac{1}{x+5}$$

$$5. \frac{1}{a} = \frac{-6}{a^2+5}$$

$$6. \frac{-1}{x^2-3x} = \frac{1}{x} + \frac{x}{x-3}$$

VII Solve the absolute value equations.

$$1. |5-2z|-1=8$$

$$2. |x+5|-7=-2$$

$$3. |5x-1|=-2$$

$$4. \left| \frac{1}{2}x - \frac{3}{4} \right| = \frac{1}{4}$$

$$5. |y-1|=|7+y|$$

VIII Solve the exponential equations.

$$1. 10^x = 1000$$

$$2. 10^{3x+5} = 100$$

$$3. 2^{x+1} = \frac{1}{8}$$

$$4. 3^{x^2} (9^x) = \frac{1}{3}$$

$$5. 2^{x^2} (4^{2x}) = \frac{1}{8}$$

IX Solve the logarithmic equations.

$$1. \log_2(x+5) = \log_2(1-5x)$$

$$2. 2\log_3(x+1) = \log_3(4x)$$

$$3. \log_2(x+1) + \log_2(x-1) = 3$$

$$4. \ln x + \ln(2x+1) = 0$$

$$5. \ln x + \ln(x+2) = \ln 3$$

$$6. 3^{2x} = 4^{x+1}$$

X Solve the radical equations.

$$1. 4\sqrt{2y-1}-2=0$$

$$2. \sqrt{2x+1}+5=8$$

$$3. \sqrt{5x-1}-2\sqrt{x+1}=0$$

$$4. \sqrt{x^2+9}+x+1=0$$

$$5. \sqrt[3]{3x+2}+4=6$$

$$6. \sqrt[4]{w^2+7}=2$$

XI Solving Inequalities

Solve the following inequalities. Express the answer graphically and using interval notation.

$$1. \frac{3}{5}x + 4 \leq -2$$

$$2. 3(x+3) \geq 5(x-1)$$

$$3. 3(x+2)-6 > -2(x-3)+14$$

$$4. 2 \leq 3x-10 \leq 5$$

$$5. |4x+1| \leq 6$$

$$6. |4x+3|+2 > 9$$

$$7. \left| \frac{x+5}{3} \right| \geq 5$$

$$8. |5-2x| < 15$$

$$9. 3x^2-11x-4 < 0$$

$$10. 6x^2+5x \geq 4$$

$$11. \frac{x+2}{3-x} \geq 0$$

$$12. \frac{(x+1)(x-3)}{2x+7} \leq 0$$

XII Lines and Regions

1. Given the equation $6x + 5y = 30$, draw the graph and find the x and y intercepts and the slope.
2. Given the equation $x = -3$, draw the graph and find the x and y intercepts and the slope.
3. Given the equation $y = 4$, draw the graph and find the x and y intercepts and the slope.
4. Write, in slope-intercept form, the equation of the line that passes through the points $(4, 6)$ and $(-4, 2)$.
5. Write, in slope-intercept form, the equation of the line perpendicular to the line $4x - y = -1$ and through the point $(2, 3)$.
6. Graph the solution set of $x - y \geq 2$.
7. Graph the solution set of $-x + 3y < -6$.

XIII Relations, Domain, and Range

For each relation, state if it is a function, state the domain and the range, and sketch a graph.

1. $y = \sqrt{x+2}$
2. $y = \sqrt{x} - 2$
3. $y = \frac{x-1}{x+2}$
4. $f(x) = -|x+1| + 3$
5. $f(x) = \frac{2x-5}{x^2-9}$
6. $x = y^2 + 2$
7. $y = x^2 + 8x - 6$
8. $y = \sqrt{-x}$
9. $y = 3^x$
10. $h(x) = \frac{6x^2}{3x^2 - 2x - 1}$

XIV Exponents and Radicals

1. $\sqrt[3]{-8x^3}$
2. $5\sqrt{147} - 4\sqrt{48}$
3. $\sqrt{5}(\sqrt{15} - \sqrt{3})$
4. $\left(\frac{x^{2/3}y^{-4/3}}{x^{-5/3}}\right)^3$
5. $\sqrt[3]{\frac{40x^4}{y^9}}$
6. $\left(\frac{54a^{-6}b^2}{9a^{-3}b^8}\right)^{-2}$
7. $\frac{\sqrt[3]{27a^3}}{\sqrt[3]{2a^2b^2}}$
8. $\frac{2}{\sqrt{5} - \sqrt{3}}$
9. $\frac{x}{\sqrt{x} + 3}$

XV Complex Numbers

Perform the indicated operations and simplify.

1. $\sqrt{-16} - 4\sqrt{-9}$
2. $\sqrt{-16} \cdot \sqrt{-9}$
3. $\frac{\sqrt{-16}}{\sqrt{-9}}$
4. $(4 - 3i)(4 + 3i)$
5. $(4 - 3i)^2$
6. i^{25}
7. $\frac{3 - 2i}{4 + 5i}$

XVI Exponential Functions and Logarithms

1. Graph $f(x) = 3^x + 1$
2. Graph $g(x) = 2^{x-1}$
3. Express $8^{-2} = \frac{1}{64}$ in logarithmic form.
4. Express $\log_5 25 = 2$ in exponential form.
5. Solve $\log_2 x = 4$
6. Solve $\log_x 9 = 2$
7. Graph $h(x) = \log_3 x$
8. Use the properties of logarithms to completely expand $\log_4 \frac{3}{y}$
9. How long will it take \$850 to be worth \$1000 if it is invested at 12% interest, compounded quarterly?

XVII Systems of Equations and Matrices

1. Solve the system
$$\begin{cases} 2x + 3y = 7 \\ 6x - y = 1 \end{cases}$$
2. Solve the system
$$\begin{cases} x + 2y + 2z = 3 \\ 2x + 3y + 6z = 2 \\ -x + y + z = 0 \end{cases}$$
3. Simplify $-2 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 3 \begin{bmatrix} \frac{1}{3} & -2 \\ 1 & 6 \end{bmatrix}$
4. Multiply $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ -2 & 1 & -3 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
5. Find the determinant of $\begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}$
6. Find the inverse of $\begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix}$

XVIII Applications

1. Sam made \$10 more than twice what Pete earned in one month. If together they earned \$760, how much did each earn that month?
2. A woman burns three times as many calories running as she does when walking the same distance. If she runs 2 miles and walks 5 miles to burn a total of 770 calories, how many calories does she burn while running 1 mile?
3. A vertical pole is standing in a small lake. If one-sixth of the length of the pole is in the sand at the bottom of the lake, 25 feet of the pole is in the water, and two-thirds the total length is in the air above the water, what is the length of the pole?

XIX Conic Sections

Graph the following. Find the center, foci, and, if they exist, asymptotes.

1. $(x - 2)^2 + y^2 = 16$
2. $\frac{(x + 1)^2}{16} + \frac{(y - 2)^2}{9} = 1$
3. $\frac{(x + 1)^2}{16} - \frac{(y - 2)^2}{9} = 1$
4. $(x - 2)^2 + y = 4$

Identify the conic section and write in standard form.

1. $x^2 - 4x - 12 + y^2 = 0$
2. $9x^2 + 18x + 16y^2 - 64y = 71$
3. $9x^2 + 18x - 16y^2 - 64y = 199$
4. $x^2 + y - 4x = 0$

XX Sequences and Series

1. Write out the first four terms of the sequence whose general term is $a_n = 3n - 2$.
2. Write out the first four terms of the sequence whose general term is $a_n = n^2 - 1$.
3. Write out the first four terms of the sequence whose general term is $a_n = 2^n + 1$.
4. Find the general term for the following sequence: 2, 5, 8, 11, 14, 17, ...
5. Find the general term for the following sequence: 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, ...
6. Find the sum: $\sum_{k=0}^6 (2k - 1)$
7. Expand the sum: $\sum_{k=0}^4 \left[\binom{4}{k} x^k y^{4-k} \right]$

XXI Functions

Let $f(x) = 2x + 9$ and $g(x) = 16 - x^2$. Find the following.

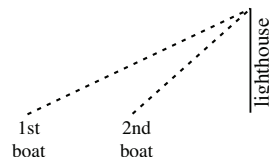
1. $f(-3) + g(2)$
2. $f(5) - g(4)$
3. $f(-1) \cdot g(-2)$
4. $\frac{f(5)}{g(2)}$
5. $(g \circ f)(-2)$
6. $f(g(x))$
7. $f^{-1}(2)$
8. $f(f^{-1}(3))$

XXII Fundamental Counting Rule, Factorials, Permutations, and Combinations

1. Evaluate $\frac{8!}{3!(8-3)!}$
2. A particular new car model is available with five choices of color, three choices of transmission, four types of interior, and two types of engine. How many difference variations of this model car are possible?
3. In a horse race, how many different finishes among the first three places are possible for a ten-horse race?
4. How many ways can a three-person subcommittee be selected from a committee of seven people?
5. How many ways can a president, vice-president, and secretary be selected from a committee of seven people?

XXIII Trigonometry

1. Graph $f(x) = \sin(x)$ through one period.
2. Graph $g(x) = \cos(2x)$ through one period.
3. A man whose eye level is 6 feet above the ground stands 40 feet from a building. The angle of elevation from eye level to the top of the building is 72° . How tall is the building?
4. A man standing at the top of a 65 meter lighthouse observes two boats. The angle of depression to the farther boat is 48° . The angle of depression to the closer boat is $54^\circ 10'$. What is the distance between the two boats?



Answers

I Factor the following polynomials.

- $15a^3b^2 - 45a^2b^3 - 60a^2b = 15a^2b(ab - 3b^2 - 4)$
- Start by factoring out the common term
 $7x^3y^3 + 21x^2y^2 - 10x^3y^2 - 30x^2y = x^2y[7xy^2 + 21y - 10xy - 30]$
Factor this result using grouping.
 $x^2y \left[\underbrace{7xy^2 + 21y - 10xy - 30} \right] = x^2y[7y(xy + 3) - 10(xy + 3)] = x^2y[(7y - 10)(xy + 3)]$
- $6x^4y^4 - 6x^3y^2 + 8xy^2 - 8 = 2(3x^3y^2 + 4)(xy^2 - 1)$
- $2x^2 - 7xy + 6y^2 = (2x - 3y)(x - 2y)$
- $y^4 + y^2 - 6 = (y^2 - 2)(y^2 + 3)$
- $7x^3 + 56y^3 = 7(x^3 + 8y^3) = 7(x + 2y)(x^2 - 2xy + 4y^2)$
- $81r^4 - 16s^4 = (9r^2 - 4s^2)(9r^2 + 4s^2) = (3r - 2s)(3r + 2s)(9r^2 + 4s^2)$
- Consider the polynomial $u^2 + 2u + 1 = (u + 1)^2$. If we let $u = x + y$, then $(x + y)^2 + 2(x + y) + 1 = (x + y + 1)^2$.

II Expand the following polynomials.

- $(x + 1)(x - 1)(x - 3) = (x^2 - 1)(x - 3) = x^3 - 3x^2 - x + 3$
- $(2x + 3y)^2 = 4x^2 + 12xy + 9y^2$
- $(\sqrt{3}x + \sqrt{3})(\sqrt{6}x - \sqrt{6}) = \sqrt{18}x^2 + \sqrt{18}x - \sqrt{18}x - \sqrt{18} = 3x^2\sqrt{2} - 3\sqrt{2}$
- $(x^2 - 2x + 3)^2 = x^4 - 4x^3 + 10x^2 - 12x + 9$
- $(x + 1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$
- $(x - 1)^6 = x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$

III Simplifying Rational Expressions

Simplify the following. Assume all variables are greater than zero.

- $3^2 + 5 - \sqrt{4} + 4^0 = 9 + 5 - 2 + 1 = 13$
- $9 \div 3 \cdot 5 - 8 \div 2 + 27 = 3 \cdot 5 - 4 + 27 = 38$
- $\sqrt{\frac{81}{x^4}} = \frac{9}{x^2}$
- $2\sqrt{18} - 5\sqrt{32} + 7\sqrt{162} = 2 \cdot 3\sqrt{2} - 5 \cdot 4\sqrt{2} + 7 \cdot 9\sqrt{2} = 49\sqrt{2}$

IV Solve the linear equations.

1. $x = 5$
2. $x = \frac{14}{5} = 2\frac{4}{5}$
3. $y = -3$
4. $x = 1$

V Solve the polynomial equations.

1. $y = \frac{8}{3}, y = -\frac{2}{3}$
2. $x = 0, x = -3, x = 5$
3. $x = \frac{1}{3}, x = \frac{-1 \pm i\sqrt{3}}{6}$
4. $x = 10, x = -4$
5. $t = \frac{-1 \pm i\sqrt{3}}{2}$
6. $x = 2, x = -1 \pm i\sqrt{3}$
7. $x = 3, x = -4$
8. $y = \frac{1 \pm \sqrt{21}}{10}$

VI Solve the rational equations.

1. $y = \frac{1}{3}$
2. Working the problem, we get $x = 3$.
However, $x = 3$ causes the denominators to be zero in the original equation. Hence, there is no solution.
3. $x = -\frac{15}{4}$
4. $x = 2$
5. $a = -1, a = -5$
6. $x = -2, x = 1$

VII Solve the absolute value equations.

1. $z = -2, z = 7$
2. $x = 0, x = -10$
3. No solution. An absolute value can not equal a negative number.
4. $x = 2, x = 1$
5. $y = -3$

VIII Solve the exponential equations.

1. $x = 3$
2. $x = -1$
3. $x = -4$
4. $x = -1$
5. $x = -1, x = -3$

IX Solve the logarithmic equations.

Rules of logarithms:

- $\log_b(x) = y \Leftrightarrow x = b^y$
- $\log_b(x \cdot y) = \log_b(x) + \log_b(y)$
- $\log_b(x \div y) = \log_b(x) - \log_b(y)$
- $\log_b(x^y) = y \log_b(x)$
- The argument and base of a logarithm must be positive.

1. $x = -\frac{2}{3}$
2. $x = 1$
3. $x = 3$; Note that $x = -3$ is not a solution since the argument of a logarithm must be positive.
4. $x = \frac{1}{2}$
5. $x = 1$
6. $x = \frac{\ln 4}{2 \ln 3 - \ln 4}$

X Solve the radical equations.

- $y = \frac{5}{8}$
- $x = 4$
- $x = 5$
- No solution. Substituting $x = 4$ into the original equation fails.
- $x = 2$
- $w = 3, w = -3$

XI Solving Inequalities

Solve the following inequalities. Express the answer graphically and using interval notation.

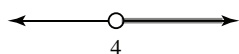
- $x \leq -10$ or Interval: $(-\infty, -10]$



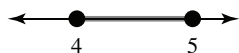
- $x \leq 7$ or Interval: $(-\infty, 7]$



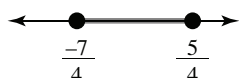
- $x > 4$ or Interval: $(4, \infty)$



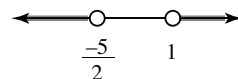
- $4 \leq x \leq 5$ or Interval: $[4, 5]$



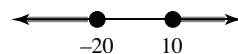
- $-\frac{7}{4} \leq x \leq \frac{5}{4}$ or Interval: $[-\frac{7}{4}, \frac{5}{4}]$



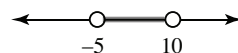
- $x < -\frac{5}{2}$ or $x > 1$ or Interval: $(-\infty, -\frac{5}{2}) \cup (1, \infty)$



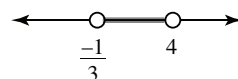
- $x \leq -20$ or $x \geq 10$ or Interval: $(-\infty, -20] \cup [10, \infty)$



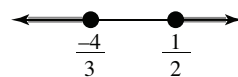
- $-5 < x < 10$ or Interval: $(-5, 10)$



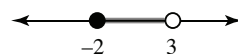
- $(-\frac{1}{3}, 4)$



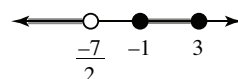
- $(-\infty, -\frac{4}{3}] \cup [\frac{1}{2}, \infty)$



- $[-2, 3)$

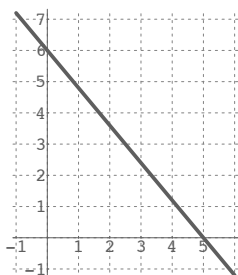


- $(-\infty, -\frac{7}{2}) \cup [-1, 3]$

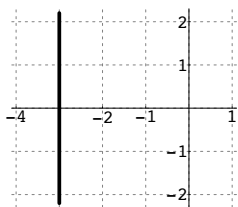


XII Lines and Regions

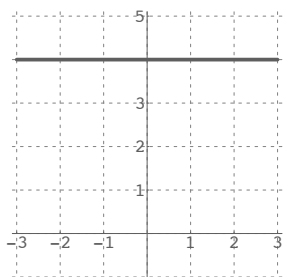
- x -intercept: $(5, 0)$
 y -intercept: $(0, 6)$
slope: $-\frac{6}{5}$



- x -intercept: $(-3, 0)$
 y -intercept: none
slope: none



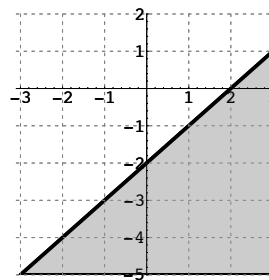
- x -intercept: none
 y -intercept: $(0, 4)$
slope: 0



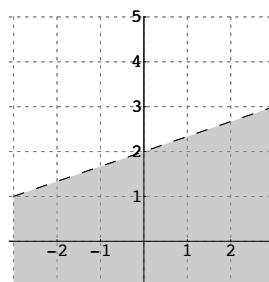
- $y = \frac{1}{2}x + 4$

- $y = -\frac{1}{4}x + \frac{7}{2}$

- $x - y \geq 2$



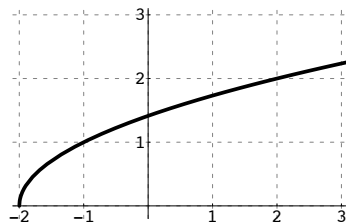
- $-x + 3y < -6$



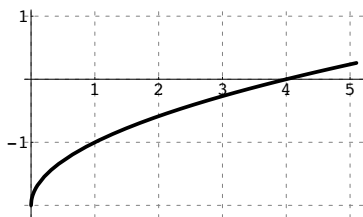
XIII Relations, Domain, and Range

For each relation, state if it is a function, state the domain and the range, and sketch a graph.

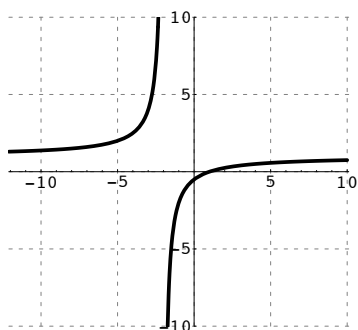
- $y = \sqrt{x + 2}$
Domain: $[-2, \infty)$
Range: $[0, \infty)$



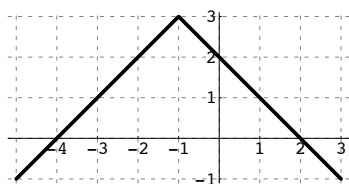
2. $y = \sqrt{x} - 2$
 Domain: $[0, \infty)$
 Range: $[-2, \infty)$



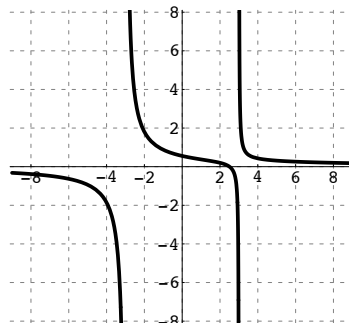
3. $y = \frac{x - 1}{x + 2}$
 Domain: All real numbers except -2
 Range: All real numbers except 1



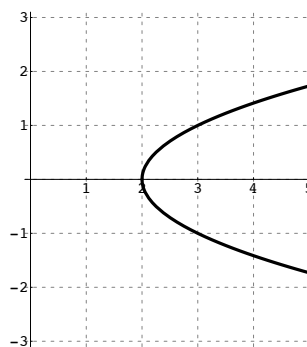
4. $f(x) = -|x + 1| + 3$
 Domain: $(-\infty, \infty)$
 Range: $(-\infty, 3]$



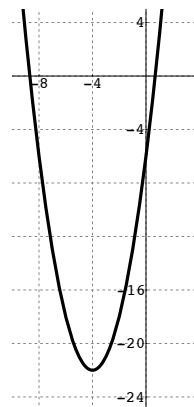
5. $f(x) = \frac{2x - 5}{x^2 - 9}$
 Domain: All real numbers except ± 3
 Range: $(-\infty, \infty)$



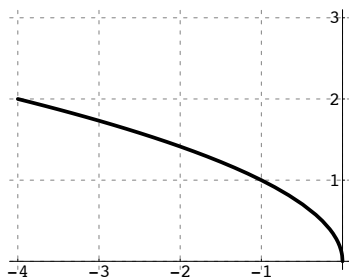
6. $x = y^2 + 2$
 Domain: $[2, \infty)$
 Range: $(-\infty, \infty)$



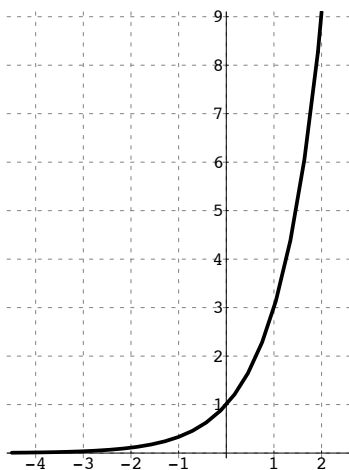
7. $y = x^2 + 8x - 6$
 Domain: $(-\infty, \infty)$
 Range: $[-22, \infty)$



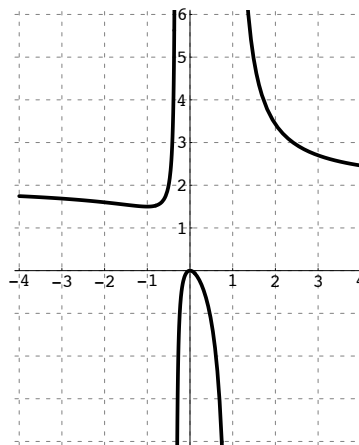
8. $y = \sqrt{-x}$
 Domain: $(-\infty, 0]$
 Range: $[0, \infty)$



9. $y = 3^x$
 Domain: $(-\infty, \infty)$
 Range: $(0, \infty)$



10. $h(x) = \frac{6x^2}{3x^2 - 2x - 1}$
 Domain: All real numbers except $-\frac{1}{3}$ and 1
 Range: $(-\infty, 0] \cup \left(\frac{3}{2}, \infty\right)$



XIV Exponents and Radicals

- $\sqrt[3]{-8x^3} = -2x$
- $5\sqrt{147} - 4\sqrt{48} = 19\sqrt{3}$
- $\sqrt{5}(\sqrt{15} - \sqrt{3}) = 5\sqrt{3} - \sqrt{15}$
- $\left(\frac{x^{2/3}y^{-4/3}}{x^{-5/3}}\right)^3 = \frac{x^7}{y^4}$
- $\sqrt[3]{\frac{40x^4}{y^9}} = \frac{2x\sqrt[3]{5x}}{y^3}$
- $\left(\frac{54a^{-6}b^2}{9a^{-3}b^8}\right)^{-2} = \frac{a^6b^{12}}{36}$
- $\frac{\sqrt[3]{27a^3}}{\sqrt[3]{2a^2b^2}} = \frac{3\sqrt[3]{4ab}}{2b}$
- $\frac{2}{\sqrt{5} - \sqrt{3}} = \sqrt{5} + \sqrt{3}$
- $\frac{x}{\sqrt{x} + 3} = \frac{x(\sqrt{x} - 3)}{x - 9}$

XV Complex Numbers

Perform the indicated operations and simplify.

$$1. \sqrt{-16} - 4\sqrt{-9} = -8i$$

$$2. \sqrt{-16} \cdot \sqrt{-9} = -12$$

$$3. \frac{\sqrt{-16}}{\sqrt{-9}} = \frac{4}{3}$$

$$4. (4 - 3i)(4 + 3i) = 25$$

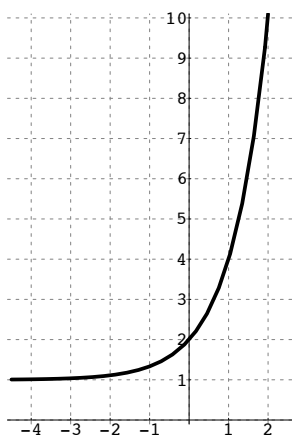
$$5. (4 - 3i)^2 = 7 - 24i$$

$$6. i^{25} = i$$

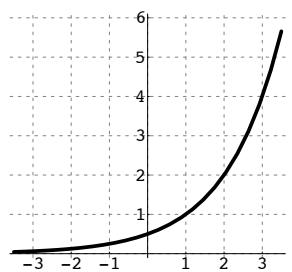
$$7. \frac{3 - 2i}{4 + 5i} = \frac{2 - 23i}{41}$$

XVI Exponential Functions and Logarithms

$$1. f(x) = 3^x + 1$$



$$2. g(x) = 2^{x-1}$$



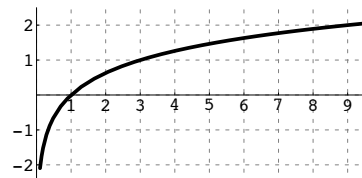
$$3. \log_8 \left(\frac{1}{64} \right) = -2$$

$$4. 5^2 = 25$$

$$5. 16$$

$$6. 3$$

$$7. h(x) = \log_3 x$$



$$8. \log_4 3 - \log_4 y$$

$$9. \text{ Use } A = P \left(1 + \frac{r}{k} \right)^{kt}$$

$$1000 = 850 \left(1 + \frac{0.12}{4} \right)^{4t}$$

$$t \approx 1.37 \text{ years.}$$

XVII Systems of Equations and Matrices

$$1. \left(\frac{1}{2}, 2, -\frac{3}{4} \right)$$

$$2. \begin{bmatrix} -\frac{3}{5}k + \frac{2}{5} \\ -\frac{8}{5}k + \frac{2}{5} \\ k \end{bmatrix} \text{ for } k \in \text{natural numbers.}$$

$$3. \begin{bmatrix} -5 & -8 \\ 5 & 14 \end{bmatrix}$$

$$4. \begin{bmatrix} -1 & 0 & 2 \\ 2 & 4 & 0 \\ 1 & -2 & -5 \end{bmatrix}$$

$$5. 5$$

$$6. \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

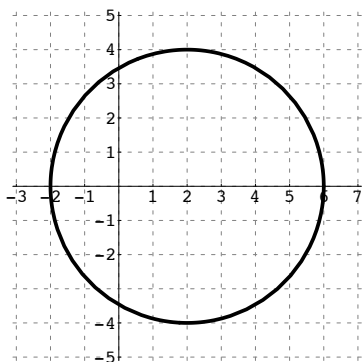
XVIII Applications

- Let x = money Pete earns,
 $2x + 10$ = money Sam earns
 $(2x + 10) + x = 760 \Rightarrow x = 250$ so Pete earns \$250 and Sam earns \$510.
- Let x = calories burned walking,
 $3x$ = calories burned running
 $2(3x) + 5x = 770 \Rightarrow x = 70$ so 210 calories burned walking and 210 calories burned running.
- Let x = length of the pole.
 $\frac{2}{3}x + 25 + \frac{1}{6}x = x \Rightarrow x = 150$ feet.

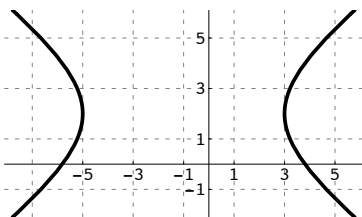
XIX Conic Sections

Graph the following. Find the center, foci, and asymptotes if possible.

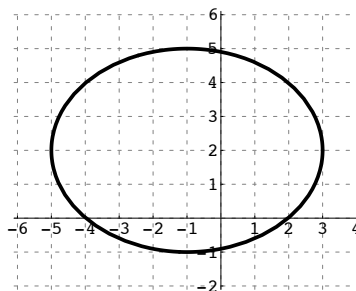
- $(x - 2)^2 + y^2 = 16$
 Circle: center $(2, 0)$, radius 4.



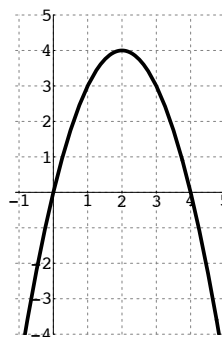
- $\frac{(x + 1)^2}{16} + \frac{(y - 2)^2}{9} = 1$
 Ellipse: center $(-1, 2)$, foci $(-1 \pm \sqrt{7}, 2)$



- $\frac{(x + 1)^2}{16} - \frac{(y - 2)^2}{9} = 1$
 Hyperbola: center $(-1, 2)$,
 foci $(-6, 2), (4, 2)$,
 asymptotes $y = \frac{3}{4}x - \frac{3}{4}, y = -\frac{3}{4}x + \frac{5}{4}$



- $(x - 2)^2 + y = 4$
 Parabola: vertex $(2, 4)$, focus $(2, \frac{15}{4})$,
 directrix $y = \frac{17}{4}$



Identify the conic section and write in standard form.

- Circle: $(x - 2)^2 + y^2 = 16$
- Ellipse: $\frac{(x + 1)^2}{16} + \frac{(y - 2)^2}{9} = 1$
- Hyperbola: $\frac{(x + 1)^2}{16} - \frac{(y - 2)^2}{9} = 1$
- Parabola: $y = -(x - 2)^2 + 4$

XX Sequences and Series

- 1, 4, 7, 10
- 0, 3, 8, 15
- 3, 5, 9, 17
- $a_n = 3n - 1$
- $a_n = 4 \cdot \left(\frac{1}{2}\right)^{n-1} = 2^{3-n}$
- $-1 + 1 + 3 + 5 + 7 + 9 + 11 = 35$
- $y^4 + 4xy^3 + 6x^2y^2 + 4x^3y + x^4$

XXI Functions

Let $f(x) = 2x + 9$ and $g(x) = 16 - x^2$. Find the following.

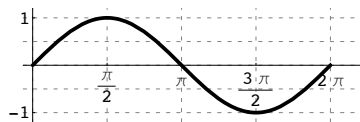
- $f(-3) + g(2) = 3 + 12 = 15$
- $f(5) - g(4) = 19 - 0 = 19$
- $f(-1) \cdot g(-2) = 7 \cdot 12 = 84$
- $\frac{f(5)}{g(5)} = \frac{19}{-9} = -\frac{19}{9}$
- $(g \circ f)(-2) = g(f(-2)) = g(5) = -9$
- $f(g(x)) = f(16 - x^2) = 2(16 - x^2) + 9 = -2x^2 + 41$
- $f^{-1}(x) = \frac{x-9}{2}; f^{-1}(2) = \frac{2-9}{2} = -\frac{7}{2}$
- $f(f^{-1}(3)) = 3$

XXII Fundamental Counting Rule, Factorials, Permutations, and Combinations

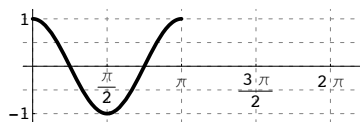
- 56
- 120
- 720
- 35
- 210

XXIII Trigonometry

- Graph $f(x) = \sin(x)$ through one period.



- Graph $g(x) = \cos(2x)$ through one period.



- $x = 6 + 40 \tan 72^\circ \approx 129.1$ ft.
- Distance between the boats = $65 \tan 42^\circ - 65 \tan 35^\circ 50' \approx 11.59$ meters.