

Ohlone College Placement Center Elementary Algebra Study Guide

Developed by Aims Community College

I Order of Operations

1. $3^2 + 5 - \sqrt{4} + 4^0$
2. $(5 + 1)(4 - 2) - 3$
3. $3 \cdot 7^2$
4. $2(7 + 3)^2$
5. $49 \div 7 - 2 \cdot 2$
6. $9 \div 3 \cdot 5 - 8 \div 2 + 27$
7. $3 + 2(5) - |-7|$
8. $\frac{5 \cdot 5 - 4(4)}{2^2 - 1}$
9. $\frac{4^2 - 5^2}{(4 - 5)^2}$
10. -5^2

II Scientific Notation

Write the following in scientific notation.

1. 350,000,000
2. 0.00000000000000523
3. 120,500,000,000,000,000,000

Write in expanded form.

1. 6.02×10^4
2. 3.0×10^8
3. 1.819×10^{-9}

Simplify and write in scientific notation.

1. $(3 \times 10^3)(5 \times 10^6)$
2. $\frac{6 \times 10^9}{3 \times 10^4}$
3. $(3 \times 10^{-4})^2$
4. $\frac{(3.2 \times 10^5)(2 \times 10^{-3})}{2 \times 10^{-5}}$

III Substitution

Find each value if $x = 3$, $y = -4$, and $z = 2$.

1. $xyz - 4z$
2. $2x - y$
3. $x(y - 3z)$
4. $\frac{5x - z}{xy}$
5. $3y^2 - 2x + 4z$

IV Linear Equations in One Variable

Solve the following for x .

1. $6x - 48 = 6$
2. $\frac{2}{3}x - 5 = x - 3$
3. $50 - x - (3x + 2) = 0$
4. $8 - 4(x - 1) = 2 + 3(4 - x)$

V Formulas

Solve for the indicated variable.

1. Solve $PV = nRT$ for T .
2. Solve $\frac{x}{2} + \frac{y}{5} = 1$ for y .
3. Solve $y = 3x + 2$ for x .
4. Solve $y = hx + 4x$ for x .
5. Solve $C = 2\pi r$ for r .

VI Word Problems

Use algebra to solve the following word problems.

1. One number is 5 more than twice another number. The sum of the numbers is 35. Find the numbers.
2. Ms. Jones invested \$18,000 in two accounts. One account pays 6% simple interest and the other pays 8%. Her total interest for the year was \$1290. How much did she have in each account?
3. How many liters of a 40% solution and a 16% solution must be mixed to obtain 20 liters of a 22% solution?
4. Sheila bought burgers and fries for her children and some friends. The burgers cost \$2.05 each and the fries cost \$0.85 each. She bought a total of 14 items, for a total cost of \$19.10. How many of each did she buy?

VII Inequalities

Solve and graph the solution on a number line.

1. $2x - 7 \geq 3$
2. $-5(2x + 3) < 2x - 3$
3. $3(x - 4) - (x + 1) \leq -12$

VIII Exponents and Polynomials

Simplify and write answers with positive exponents.

1. $(3x^2 - 5x - 6) - (5x^2 + 4x + 4)$
2. $(-a^5b^7c^9)^4$
3. $(3x^0y^5z^6)(-2xy^3z^{-2})$
4. $(4x^2y^6z)^2(-x^{-2}y^3z^4)^6$
5. $\frac{26a^2b^{-5}c^9}{-4a^{-6}bc^9}$
6. $\frac{(2a^{-5}b^4c^3)^{-2}}{(3a^3b^{-7}c^3)^2}$
7. $\frac{24x^4 - 32x^3 + 16x^2}{8x^2}$
8. $(x^2 - 5x)(2x^3 - 7)$
9. $(5a + 6)^2$

IX Factoring

1. $x^2 + 5x - 6$
2. $x^2 - 5x - 6$
3. $4x^2 - 36$
4. $x^2 + 4$
5. $64x^4 - 4y^4$
6. $8x^3 - 27$
7. $49y^2 + 84y + 36$
8. $12x^2 + 12x + 3$

X Quadratic Equations

Solve the following.

1. $a^2 + 5a + 6 = 0$
2. $9x^2 - 81 = 0$
3. $25x^2 - 6 = 30$

$$4. 3x^2 - 5x - 2 = 0$$

$$5. (3x + 2)^2 = 16$$

$$6. r^2 - 2r - 4 = 0$$

XI Rational Expressions

Perform the following operations and simplify the results.

$$1. \frac{4}{a+2} + \frac{2a}{a+2}$$

$$2. \frac{4}{2a-2} + \frac{3a}{a^2-a}$$

$$3. \frac{3}{x^2-1} - \frac{4}{x^2+3x+2}$$

$$4. \frac{6x-18}{3x^2+2x-8} \cdot \frac{12x-16}{4x-12}$$

$$5. \frac{16-x^2}{x^2+2x-8} \div \frac{x^2-2x-8}{4-x^2}$$

$$6. \frac{\frac{2}{x} - \frac{1}{y}}{\frac{1}{xy}}$$

$$7. \frac{x^3-1}{x-1}$$

Solve the following.

$$1. \frac{3}{k} + 1 = \frac{3+k}{2k}$$

$$2. \frac{2}{x-1} + \frac{1}{x+1} = \frac{5}{4}$$

$$3. \frac{5-x}{x} + \frac{3}{4} = \frac{7}{x}$$

XII Graphing

Graph each equation on a set of xy coordinate axes.

$$1. 3x - 2y = 6$$

$$2. x = -3$$

$$3. y = 4$$

$$4. y = -\frac{2}{3}x + 5$$

$$5. y = |x - 3|$$

$$6. y = -x^2 + 2$$

$$7. y = \sqrt{x+2}$$

XIII Systems of Equations

Solve the following systems of equations.

$$1. \begin{cases} 2x - 3y = -12 \\ x - 2y = -9 \end{cases}$$

$$2. \begin{cases} 4x + 6y = 10 \\ 2x + 3y = 5 \end{cases}$$

$$3. \begin{cases} x + 2y = 5 \\ x + 2y = 7 \end{cases}$$

$$4. \begin{cases} 2x - 3y = -4 \\ y = -2x + 4 \end{cases}$$

XIV Radicals

Simplify the following using the rules of radicals. Rationalize all denominators. All variables represent positive numbers.

$$1. (\sqrt{8})(\sqrt{10})$$

$$2. \sqrt[4]{\frac{81}{x^4}}$$

$$3. \sqrt{\frac{4}{3}}$$

$$4. \sqrt{\frac{12}{18}} \sqrt{\frac{15}{40}}$$

$$5. \sqrt[3]{24x^3y^6}$$

$$6. 2\sqrt{18} - 5\sqrt{32} + 7\sqrt{162}$$

$$7. \frac{\sqrt{3}}{5 - \sqrt{3}}$$

$$8. (2\sqrt{3} + 5\sqrt{2})(3\sqrt{3} - 4\sqrt{2})$$

Answers

I Order of Operations

When working with parentheses $()$, exponents a^x , \times , \div , $-$, and $+$, one must remember the order of operations. First, work inside any set of parentheses. Next, use exponents and radicals. Third, use multiplication and division. Finally, use addition and subtraction. As you use these operations, always work from left to right.

1. $3^2 + 5 - \sqrt{4} + 4^0 = 9 + 5 - 2 + 1 = 14 - 2 + 1 = 12 + 1 = 13$

2. $(5 + 1)(4 - 2) - 3 = (6)(2) - 3 = 12 - 3 = 9$

3. $3 \cdot 7^2 = 3 \cdot 49 = 147$

4. $2(7 + 3)^2 = 2(10)^2 = 2 \cdot 100 = 200$

5. $49 \div 7 - 2 \cdot 2 = 7 - 4 = 3$

6. $9 \div 3 \cdot 5 - 8 \div 2 + 27 = 3 \cdot 5 - 4 + 27 = 15 - 4 + 27 = 38$

7. $3 + 2(5) - |-7| = 3 + 10 - 7 = 6$

8. $\frac{5 \cdot 5 - 4(4)}{2^2 - 1} = \frac{25 - 16}{4 - 1} = \frac{9}{3} = 3$

9. $\frac{4^2 - 5^2}{(4 - 5)^2} = \frac{16 - 25}{(-1)^2} = \frac{-9}{1} = -9$

10. $-5^2 = -25$ (The negative is not squared.)

II Scientific Notation

All numbers in scientific notation have the form of a single digit, followed by the decimal point, then the rest of the digits, then multiplied by a power of ten.

Write the following in scientific notation.

1. $350,000,000 = 3.5 \times 10^8$

2. $0.0000000000000523 = 5.23 \times 10^{-14}$

3. $120,500,000,000,000,000,000 = 1.205 \times 10^{20}$

Write in expanded form.

1. $6.02 \times 10^4 = 60,200$

2. $3.0 \times 10^8 = 300,000,000$

3. $1.819 \times 10^{-9} = 0.00000001819$

Simplify and write in scientific notation.

1. $(3 \times 10^3)(5 \times 10^6) = 15 \times 10^9 = 1.5 \times 10^{10}$
2. $\frac{6 \times 10^9}{3 \times 10^4} = 2 \times 10^5$
3. $(3 \times 10^{-4})^2 = 9 \times 10^{-8}$
4. $\frac{(3.2 \times 10^5)(2 \times 10^{-3})}{2 \times 10^{-5}} = 3.2 \times 10^7$

III Substitution

Find each value if $x = 3$, $y = -4$, and $z = 2$.

1. $xyz - 4z = 3(-4)(2) - 4(2) = -24 - 8 = -32$
2. $2x - y = 2(3) - (-4) = 6 + 4 = 10$
3. $x(y - 3z) = 3(-4 - 3 \cdot 2) = 3(-4 - 6) = 3(-10) = -30$
4. $\frac{5x - z}{xy} = \frac{5(3) - 2}{3(-4)} = -\frac{13}{12}$
5. $3y^2 - 2x + 4z = 3(-4)^2 - 2(3) + 4(2) = 3(16) - 6 + 8 = 50$

IV Linear Equations in One Variable

Solve the following for x .

1. $6x - 48 = 6 \Rightarrow 6x = 54 \Rightarrow x = 9$
2. $\frac{2}{3}x - 5 = x - 3 \Rightarrow -2 = \frac{1}{3}x \Rightarrow x = -6$
3. $50 - x - (3x + 2) = 0 \Rightarrow 48 = 4x \Rightarrow x = 12$
4. $8 - 4(x - 1) = 2 + 3(4 - x) \Rightarrow 12 - 4x = 14 - 3x \Rightarrow -x = 2 \Rightarrow x = -2$

V Formulas

Solve for the indicated variable.

1. $T = \frac{PV}{nR}$.
2. $\frac{x}{2} + \frac{y}{5} = 1 \Rightarrow \frac{y}{5} = 1 - \frac{x}{2} = \frac{2-x}{2} \Rightarrow y = \frac{5(2-x)}{2}$ or $y = \frac{-5x}{2} + 5$
3. $y = 3x + 2 \Rightarrow y - 2 = 3x \Rightarrow x = \frac{y-2}{3}$
4. $y = hx + 4x \Rightarrow y = x(h+4) \Rightarrow x = \frac{y}{h+4}$
5. $C = 2\pi r \Rightarrow r = \frac{C}{2\pi}$

VI Word Problems

Use algebra to solve the following word problems.

1. The two numbers are 10 and 25.

Let x represent the first number. Then the second number is five more than twice x or $5 + 2x$. The sum of these numbers is thirty-five, giving the equation $x + (5 + 2x) = 35$. Solving gives $x = 10$ so the other number is $5 + 2x = 5 + 2(10) = 25$.

2. The correct answer is \$7500 at 6% and \$10,500 at 8%.

Let x be the amount invested at 6%. Since a total of \$18,000 is invested, the amount invested at 8% must be $18,000 - x$. To calculate the interest, multiply the interest rate by the amount invested. Adding together the interest from both accounts gives $0.06x + 0.08(18,000 - x) = 1290$. Solving gives $x = 7500$.

3. The correct answer is 5 liters at 40% solution and 15 liters of 16% solution.

Let x be the amount of 40% solution. Since a total of 20 liters is needed, the amount of 16% solution must be $20 - x$. To calculate the amount of chemical in the solution, multiply the percentage by the amount. Adding together the chemicals from both solutions gives $0.40x + 0.16(20 - x) = 0.22(20)$. Solving gives $x = 5$.

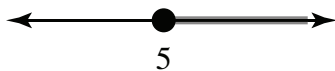
4. The correct answer is 6 hamburgers and 8 fries.

Let x be the number of hamburgers. Since a total of 14 items are purchased, the number of fries purchased must be $14 - x$. To calculate the amount of money spent, multiply the number of items purchased by the cost of the item. Adding together the costs from the hamburgers and fries gives $2.05x + 0.85(14 - x) = 19.10$. Solving gives $x = 6$.

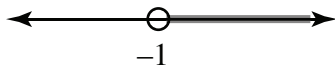
VII Inequalities

Solve and graph on a number line.

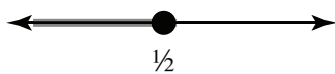
1. $2x - 7 \geq 3 \Rightarrow x \geq 5$



2. $-5(2x + 3) < 2x - 3 \Rightarrow x > 1$



3. $3(x - 4) - (x + 1) \leq -12 \Rightarrow x \leq \frac{1}{2}$



VIII Exponents and Polynomials

Simplify and write answers with positive exponents.

- $(3x^2 - 5x - 6) - (5x^2 + 4x + 4) = -2x^2 - 9x - 10$
- $(-a^5b^7c^9)^4 = a^{20}b^{28}c^{36}$
- $(3x^0y^5z^6)(-2xy^3z^{-2}) = -6xy^8z^4$
- $(4x^2y^6z)^2(-x^{-2}y^3z^4)^6 = (16x^4y^{12}z^2)(x^{-12}y^{18}z^{24}) = 16x^{-8}y^{30}z^{26} = \frac{16y^{30}z^{26}}{x^8}$
- $\frac{26a^2b^{-5}c^9}{-4a^{-6}bc^9} = \frac{-13a^8}{2b^6}$
- $\frac{(2a^{-5}b^4c^3)^{-2}}{(3a^3b^{-7}c^3)^2} = \frac{2^{-2}a^{10}b^{-8}c^{-6}}{9a^6b^{-14}c^6} = \frac{a^{10}b^{14}}{4 \cdot 9a^6b^8c^6c^6} = \frac{a^4b^6}{36c^{12}}$
- $\frac{24x^4 - 32x^3 + 16x^2}{8x^2} = 3x^2 - 4x + 2$
- $(x^2 - 5x)(2x^3 - 7) = 2x^5 - 10x^4 - 7x^2 + 35x$
- $(5a + 6)^2 = (5a + 6)(5a + 6) = 25a^2 + 30a + 30a + 36 = 25a^2 + 60a + 36$

IX Factoring

- $x^2 + 5x - 6 = (x - 1)(x + 6)$
- $x^2 - 5x - 6 = (x + 1)(x - 6)$
- $4x^2 - 36 = 4(x^2 - 9) = 4(x - 3)(x + 3)$
- $x^2 + 4$ is not factorable.
- $64x^4 - 4y^4 = 4(16x^4 - y^4) = 4(4x^2 + y^2)(4x^2 - y^2) = 4(4x^2 + y^2)(2x + y)(2x - y)$
- $8x^3 - 27 = (2x - 3)(4x^2 + 6x + 9)$
- $49y^2 + 84y + 36 = (7y + 6)^2$
- $12x^2 + 12x + 3 = 3(4x^2 + 4x + 1) = 3(2x + 1)^2$

X Quadratic Equations

- $a^2 + 5a + 6 = (a + 3)(a + 2) = 0 \Rightarrow a = -3$ or $a = -2$
- $9x^2 - 81 = 9(x^2 - 9) = 9(x + 3)(x - 3) = 0 \Rightarrow x = -3$ or $x = 3$
- $25x^2 - 6 = 30 \Rightarrow 25x^2 - 36 = 0 \Rightarrow (5x - 6)(5x + 6) = 0 \Rightarrow x = \frac{6}{5}$ or $x = -\frac{6}{5}$

$$4. 3x^2 - 5x - 2 = (3x + 1)(x - 2) = 0 \Rightarrow x = \frac{-1}{3} \text{ or } x = 2$$

$$5. (3x + 2)^2 = 16 \Rightarrow 9x^2 + 12x - 12 = 0 \Rightarrow 3(3x - 2)(x + 2) = 0 \Rightarrow x = \frac{2}{3} \text{ or } x = -2$$

$$6. r^2 - 2r - 4 = 0 \Rightarrow r = 1 + \sqrt{5} \text{ or } r = 1 - \sqrt{5}$$

XI Rational Expressions

Perform the following operations and simplify the results.

$$1. \frac{4}{a+2} + \frac{2a}{a+2} = \frac{2a+4}{a+2} = \frac{2(a+2)}{a+2} = 2$$

$$2. \frac{4}{2a-2} + \frac{3a}{a^2-a} = \frac{4}{2(a-1)} \cdot \frac{a}{a} + \frac{3a}{a(a-1)} \cdot \frac{2}{2} = \frac{4a+6a}{2a(a-1)} = \frac{10a}{2a(a-1)} = \frac{5}{a-1}$$

$$3. \frac{3}{x^2-1} - \frac{4}{x^2+3x+2} = \frac{3}{(x-1)(x+1)} \cdot \frac{x+2}{x+2} - \frac{4}{(x+1)(x+2)} \cdot \frac{x-1}{x-1} = \frac{3x+6-4x+4}{(x-1)(x+1)(x+2)} = \frac{10-x}{(x-1)(x+1)(x+2)}$$

$$4. \frac{6x-18}{3x^2+2x-8} \cdot \frac{12x-16}{4x-12} = \frac{6(x-3)}{(3x-4)(x+2)} \cdot \frac{4(3x-4)}{4(x-3)} = \frac{6}{x+2}$$

$$5. \frac{16-x^2}{x^2+2x-8} \div \frac{x^2-2x-8}{4-x^2} = \frac{16-x^2}{x^2+2x-8} \cdot \frac{4-x^2}{x^2-2x-8} = \frac{(4-x)(4+x)}{(x+4)(x-2)} \cdot \frac{(2+x)(2-x)}{(x-4)(x+2)} = 1$$

$$6. \frac{\frac{2}{x} - \frac{1}{y}}{\frac{1}{xy}} \cdot \frac{xy}{1} = \frac{2y-x}{1} = 2y-x$$

$$7. \frac{x^3-1}{x-1} = \frac{(x-1)(x^2+x+1)}{x-1} = x^2+x+1$$

Solve the following.

$$1. \frac{3}{k} + 1 = \frac{3+k}{2k}; \text{ Multiplying by } 2k \text{ gives } 2(3) + 2k = 3+k \Rightarrow k = -3$$

$$2. \frac{2}{x-1} + \frac{1}{x+1} = \frac{5}{4}; \text{ Multiplying by } 4(x+1)(x-1) \text{ gives}$$

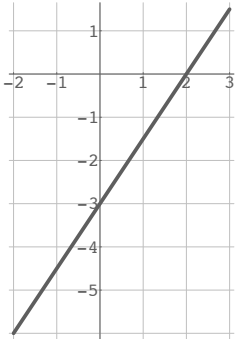
$$4(x+1)(2) + 4(x-1) = 5(x-1)(x+1) \Rightarrow 12x+4 = 5x^2-5 \Rightarrow$$

$$5x^2-12x-9 = 0 \Rightarrow x = \frac{-3}{5} \text{ or } x = 3.$$

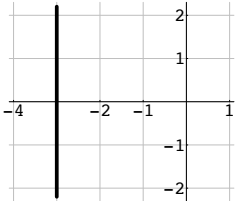
$$3. \frac{5-x}{x} + \frac{3}{4} = \frac{7}{x}; \text{ Multiplying by } 4x \text{ gives } 4(5-x) + 3x = 4(7) \Rightarrow -x = 8 \Rightarrow x = -8$$

XII Graphing

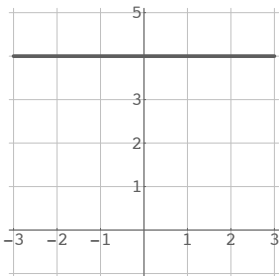
1. $3x - 2y = 6$



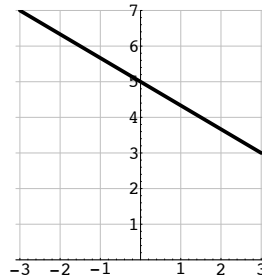
2. $x = -3$



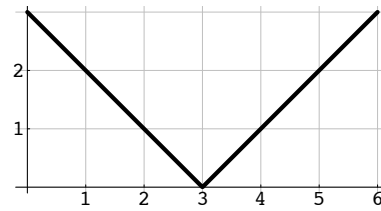
3. $y = 4$



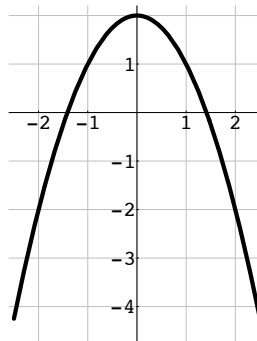
4. $y = -\frac{2}{3}x + 5$



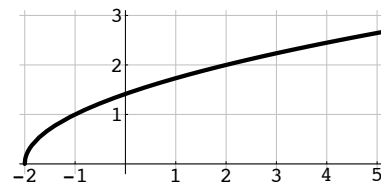
5. $y = |x - 3|$



6. $y = -x^2 + 2$



7. $y = \sqrt{x + 2}$



XIII Systems of Equations

Solve the following systems of equations.

1. The answer is $(3, 6)$ or $x = 3, y = 5$;
Multiply the second equation by 2 and add the two equations together to give $y = 6$.
Substitute $y = 6$ into one of the original equations to get $x = 3$.
2. Since these are the same line, there are an infinite number of solutions.
3. The lines are parallel so there is no solution.
4. $x = 1, y = 2$

XIV Radicals

Simplify the following using the rules of radicals. Rationalize all denominators. All variables represent positive numbers.

1. $(\sqrt{8})(\sqrt{10}) = \sqrt{80} = \sqrt{16 \cdot 5} = 4\sqrt{5}$
2. $\sqrt[4]{\frac{81}{x^4}} = \frac{3}{x}$
3. $\sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
4. $\sqrt{\frac{12}{18}} \sqrt{\frac{15}{40}} = \sqrt{\frac{2}{3}} \sqrt{\frac{3}{8}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$
5. $\sqrt[3]{24x^3y^6} = \sqrt[3]{3 \cdot 8x^3y^6} = 2xy^2 \sqrt[3]{3}$
6. $2\sqrt{18} - 5\sqrt{32} + 7\sqrt{162} = 2 \cdot 3\sqrt{2} - 5 \cdot 4\sqrt{2} + 7 \cdot 9\sqrt{2} = 49\sqrt{2}$
7. $\frac{\sqrt{3}}{5 - \sqrt{3}} \cdot \frac{5 + \sqrt{3}}{5 + \sqrt{3}} = \frac{5\sqrt{3} + 3}{25 - 3} = \frac{5\sqrt{3} + 3}{22}$
8. $(2\sqrt{3} + 5\sqrt{2})(3\sqrt{3} - 4\sqrt{2}) = 6 \cdot 3 - 8\sqrt{6} + 15\sqrt{6} - 20 \cdot 2 = 7\sqrt{6} - 22$