1. Describe a situation in which the limit \( \lim_{x \to a} f(x) \) exists but \( f(x) \) is not continuous at \( x = a \).

**Solution:** There are many possible answers. However, your solution should involve a hole at \((a, f(a))\).

2. Describe a situation in which the limit \( \lim_{x \to a} f(x) = \infty \) and \( f(x) \) is not continuous at \( x = a \).

**Solution:** There are many possible answers. However, your solution should involve a vertical asymptote at \((a, f(a))\) in which the graph approaches positive infinity on both sides of the asymptote.

3. Draw a function for which the limit \( \lim_{x \to \infty} f(x) = 7 \) and \( \lim_{x \to -\infty} f(x) = -7 \).

**Solution:** There are many possible answers. The graph should include two horizontal asymptotes.

4. True or False
   (a) If a calculation shows \( \lim_{x \to 4} f(x) = \frac{6}{0} \), then you know that there is a vertical asymptote at \( x = 4 \).
      ■ True □ False

   (b) If a calculation shows \( \lim_{x \to 4} f(x) = \frac{0}{0} \), then you know that there is a vertical asymptote at \( x = 4 \).
      □ True ■ False

   (c) If a calculation shows \( \lim_{x \to 4} f(x) = \frac{0}{0} \), then you know that there is a hole at \( x = 4 \).
      □ True ■ False

   (d) If a calculation shows \( \lim_{x \to 4^+} f(x) = \lim_{x \to 4^-} f(x) \), then you know that the function is continuous at \( x = 4 \).
      □ True ■ False

   (e) If a calculation shows \( \lim_{x \to 4^+} f(x) = \lim_{x \to 4^-} f(x) = f(4) \), then you know that the function is continuous.
      ■ True □ False

5. Sketch a function that meets the following conditions:
   - \( f(2) = 3 \)
   - \( \lim_{x \to 2^+} f(x) = 4 \)
   - \( \lim_{x \to 2^-} f(x) = 2 \)
   - \( \lim_{x \to 1^+} f(x) = \infty \)
   - \( \lim_{x \to 1^-} f(x) = -\infty \)
   - \( \lim_{x \to 4^+} f(x) = \infty \)
   - \( \lim_{x \to 4^-} f(x) = \infty \)
6. Use algebraic methods to determine the following limits. Be sure to show your work.

(a) \( \lim_{x \to \infty} \frac{1 - x^2}{x^2 - 8x + 7} \)

**Solution:** Multiply top and bottom by \( \frac{1}{x^2} \) to get a limit of \(-1\).

(b) \( \lim_{x \to 4^+} \frac{x - 4}{\sqrt{x} - 2} \)

**Solution:** Factor the top into \((\sqrt{x} - 2)(\sqrt{x} + 2)\) and show the limit is 6.

(c) \( \lim_{x \to 2} \frac{1}{x - 2} - \frac{2}{x^2 - 2x} \)

**Solution:** Get a common denominator and then cancel the factor \((x - 2)\) to get a limit of \(\frac{1}{2}\).

(d) \( \lim_{x \to \infty} \frac{3\sqrt{x^6} + 8}{4x^2 + \sqrt{3x^4} + 1} \)

**Solution:** See page 92, number 29

7. Determine the values of \(a\) and \(b\) so that \(f(x)\) is continuous at \(x = 3\).

\[
f(x) = \begin{cases} 
5x - 2 & \text{if } x < 3 \\
a & \text{if } x = 3 \\
ax^2 + bx & \text{if } x > 3 
\end{cases}
\]

**Solution:** Plug in \(x = 3\) to find \(a = 13\) and \(b = 34.667\).

8. Use a calculator to determine the value of \( \lim_{x \to 0^+} \frac{x + \sin 4x}{2x^2 - x} \). Use at least four appropriate values of \(x\) in the table.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(f(x))</th>
<th>(\lim_{x \to 0^+} \frac{x + \sin 4x}{2x^2 - x})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Solution: You decimals should show values to the left of $x = 0$ and the limit should approach $-5$.

9. For the function $f(x) = 3x - 2$, determine the value of $\delta$ that should be used in a formal limit proof showing that $\lim_{x \to 5} f(x) = 13$ for any given value of $\epsilon$.

Solution:

$$|3x - 2 - 13| < \epsilon$$
$$|3x - 15| < \epsilon$$
$$3|x - 5| < \epsilon$$
$$|x - 5| < \frac{\epsilon}{3}$$