1. Give an example (graph, equation or English) of a function \( f(x) \) that is continuous at \( x = a \) but is not differentiable at \( x = a \).

**Solution:** There are many possible answers. However, your solution should involve a sharp point at \((a, f(a))\).

2. Use the definition of the derivative to find the derivative of \( f(x) = \sqrt{x} \) at \( x = 2 \). You cannot use the shortcut formulas.

**Solution:**
\[
\lim_{\Delta x \to 0} \frac{f(2 + \Delta x) - f(2)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sqrt{2 + \Delta x} - \sqrt{2}}{\Delta x} = \lim_{\Delta x \to 0} \frac{(\sqrt{2 + \Delta x} - \sqrt{2})(\sqrt{2 + \Delta x} + \sqrt{2})}{\Delta x(\sqrt{2 + \Delta x} + \sqrt{2})} = \lim_{\Delta x \to 0} \frac{2 + \Delta x - 2}{\Delta x(\sqrt{2 + \Delta x} + \sqrt{2})} = \lim_{\Delta x \to 0} \frac{1}{\sqrt{2 + \Delta x} + \sqrt{2}} = \frac{1}{2\sqrt{2}}
\]

3. Find the derivatives of the following. Completely simplify the results

(a) \( f(x) = \frac{3x - 2}{5x + 1} \)

**Solution:** Use the quotient rule and get \( f'(x) = \frac{13}{(5x + 1)^2} \)

(b) \( f(x) = \sec^2 5x - \tan^2 5x \)

**Solution:** Use the trig identity first to simplify the question to \( f(x) = 1 \). Then, the derivative is zero.

4. Find the equation of the tangent line to \( f(x) = x^3 - 7x^2 \) at \( x = -1 \).

**Solution:** \( f'(x) = 3x^2 - 14x \) so \( f'(-1) = 17 \). Since \( f(-1) = -8 \), the equation of the line is
\[
y - (-8) = 17(x - (-1)) \rightarrow y = 17x + 9
\]

5. Find the equation of the tangent line to \( x^2 + y^2 = 25 \) at \( x = 4 \).

**Solution:** Using implicit derivatives, find the derivative is \( 2x + 2yy' = 0 \). At \( x = 4, y = 3 \). You could also use \( y = -3 \). Thus, \( y' = \frac{x}{y} = \frac{-4}{3} \). This gives the equation of the line as
\[
y - 3 = \frac{-4}{3}(x - 4) \rightarrow y = \frac{-3}{4}x + \frac{25}{3}
\]
6. Find the derivatives of the following. Do not simplify any of the algebra after doing the derivatives.

(a) \( f(x) = 3x\sqrt{x^2 + 6x} \)

Solution:
\[
 f'(x) = 3x \left( x^2 + 6x \right)^{-1/2} \left( 1 + 6 \right) 
\]

(b) \( f(x) = \sqrt{\sin \frac{2x}{x^2 + 1}} \)

Solution:
\[
 f'(x) = \frac{1}{2} \left( \frac{2 \cos \frac{2x}{x^2 + 1}}{x^2 + 1} \right)^{-1/2} \left( \frac{(x^2 + 1)(2 \cos 2x) - \sin(2x)(2x)}{(x^2 + 1)^2} \right) 
\]

(c) \( f(x) = \ln \left( \frac{x^2 + 1}{4x + 5} \right) \)

Solution: Simplify the logarithm first so \( f(x) = \ln(x^2 + 1) + \ln(4x + 5) - \ln(x^3 - 1) \). Now take the derivative.
\[
 f'(x) = \frac{2x}{x^2 + 1} + \frac{4}{4x + 5} - \frac{15x^2}{x^3 - 1} 
\]

(d) \( f(x) = \sin \left( e^{2x} + \tan(x^3 + 1) \right) \)

Solution:
\[
 f'(x) = \cos \left( e^{2x} + \tan(x^3 + 1) \right) \left( 2e^{2x} + 3x^2 \sec^2(x^3 + 1) \right) 
\]

(e) \( f(x) = (\sin x)^{\cos 3x} \)

Solution: If \( y = (\sin x)^{\cos 3x} \), then \( \ln y = \cos 3x \ln \sin x \). Taking the derivative gives
\[
 \frac{1}{y} y' = -3 \sin 3x \ln \sin x \cos 3x \sin x \sin x 
\]

7. The curve \( f(x) = k - x^2 \) has a tangent line at the point \( (a, f(a)) \) and the tangent line passes through the point \( (0, 2k) \). What are all the possible values of \( a \)?

Solution: Since \( f'(a) = -2a \) and the tangent line passes through the point \( (0, 2k) \), the equation of the tangent line is \( y = 2k - 2ax \). Since the tangent line must pass through the point of tangency \( (a, k - a^2) \), this gives the equation \( k - a^2 = 2k - 2a^2 \). Solving this equation gives \( a = \pm \sqrt{k} \).

8. A large company makes 25,000 gadgets per year in batches of \( x \) items at a time. After analyzing setup costs to produce each batch and taking into account storage costs, it has been determined that the total cost \( C(x) \) of producing 25,000 gadgets in batches of \( x \) items at a time is given by
\[
 C(x) = 1,250,000 + \frac{125,000,000}{x} + 1.5x 
\]

(a) Write the average cost function.

Solution: The average cost function is given by
\[
 \bar{C}(x) = \frac{C(x)}{x} = \frac{1,250,000}{x} + \frac{125,000,000}{x^2} + 1.5 
\]
(b) State in English the meaning of the average cost function when $x = 5000$.

**Solution:** The average cost function gives the average cost of each item is approximately $296.50.

(c) Write the marginal cost function and simplify your answer.

**Solution:** The marginal cost is given by

$$C'(x) = -\frac{125,000,000}{x^2} + 1.5$$

(d) State in English the meaning of the marginal cost function when $x = 5000$.

**Solution:** The marginal cost is the cost of making the 5000th item.

9. An airliner passes over an airport at noon traveling at 500 mi/hr heading west. At 1:00 P.M., another airliner passes over the same airport at the same elevation traveling north at 550 mi/hr. Assuming that both airliners keep their same elevations, speeds, and directions, how fast is the distance between them changing at 2:30 P.M.?

**Solution:** See section 3.10 problem 33

10. The hands of a clock are 4 inches and 5 inches long. How fast is the distance between the tips of the hands of the clock changing at 9:05? Note that at 9:05, the angle between the hands is $\frac{47\pi}{72} \approx 2.05076$. Hint: You may want to use the Law of Cosines.

**Solution:** See section 3.10 problem 35