1. Given the function \( f(x) = 2x^3 - 3x^2 - 36x + 12 \),
   
   (a) Use the first derivative to find the location and type of the critical points.
   
   **Solution:**
   
   \[ f'(x) = 6x^2 - 6x - 36 = 0 \rightarrow x = -2, x = 3 \]
   
   Since \( f''(-2) \) is negative \((-2, 56)\) is a local maximum. Similarly, since \( f''(3) \) is positive \((3, -69)\) is a local minimum.
   
   (b) Find the location of any inflection points.
   
   **Solution:**
   
   \[ f''(x) = 12x - 6 = 0 \rightarrow x = \frac{1}{2} \]
   
   Therefore \( \left( \frac{1}{2}, -\frac{13}{2} \right) \) is the point of inflection
   
   (c) The absolute maximum and minimum points for the function on the interval \([-8, 8]\).
   
   **Solution:** Since \( f(8) = 556 \) and \( f(-8) = -916 \), the absolute minimum of \( f(x) \) on the interval \([-8, 8]\) is at \((-8, -916)\) and the absolute maximum is at \((8, 556)\).

2. True or False
   
   (a) A function \( f \) has the property that \( f'(2) = 0 \). Therefore, \( f \) has a local minimum or maximum at \( x = 2 \). \( \square \) True \( \blacksquare \) False
   
   (b) A function \( f \) has the property that \( f'(3) \) does not exist. Therefore, \( f \) has a critical point at \( x = 3 \). \( \square \) True \( \blacksquare \) False
   
   (c) If \( f \) and \( g \) are both continuous functions that increase on an interval then the product \( fg \) also increases on the interval. \( \square \) True \( \blacksquare \) False
   
   (d) There exists a function \( f \) that is continuous on \((-\infty, \infty)\) with exactly three critical points, all of which correspond to local maxima. \( \square \) True \( \blacksquare \) False

3. Find the critical points of \( f(x) = x\sqrt{x-a} \) for constant values of \( a \). You need only determine the \( x \) values, not the types or the \( y \) values.

   **Solution:**
   
   \[ f'(x) = \frac{x}{2\sqrt{x-a}} + \sqrt{x-a} = 0 \rightarrow x = \frac{2a}{3} \]
   
   \[ f'(x) = \text{undefined} \rightarrow x = a \]

4. A continuous function \( f \) has the following properties. Sketch a possible graph of \( f \).
   
   - \( f''(x) > 0 \) on \((-\infty, -1)\)
   - \( f'(-1) \) is undefined
   - \( f''(4) = 0 \)
   - \( f'(4) = 0 \)
   - \( f''(x) > 0 \) on \((1, 4)\)
   - \( f''(x) < 0 \) on \((4, \infty)\)
   - \( f'(x) < 0 \) on \((1, 4)\)
   - \( f'(x) < 0 \) on \((4, \infty)\)
5. The graph of the $f'(x)$ is shown in the picture. Sketch $f(x)$. Note that the $x$-axis forms a horizontal asymptote that is not crossed by $f'(x)$.

6. Use a derivative to approximate the change in the lateral surface area of a right circular cone of fixed height of $h = 6$ m when its radius increases from $r = 3$ m to $r = 3.05$ m. Note the lateral surface area is given by $S = \pi r \sqrt{r^2 + h^2}$.

**Solution:**

Using differentials gives

$$dS = \pi \left[ \sqrt{r^2 + h^2} + \frac{r^2}{\sqrt{r^2 + h^2}} \right] dr$$

Substituting $r = 3$, $h = 6$, and $dr = 0.05$ gives

$$dS = \pi \left[ \sqrt{3^2 + 6^2} + \frac{3^2}{\sqrt{3^2 + 6^2}} \right] (0.05)$$

$$dS \approx 1.26 \text{ m}^2$$

7. A box is constructed such that its length is 6 inches more than twice its width and the sum of the three dimensions of the box is 63 inches. What is one of the dimensions of the box of maximum volume?
Note that the results will be messy decimals. You do not need to find all three dimensions.

**Solution:** You know \( L = 6 + 2W \) and \( L + W + H = 63 \). Substituting these into the volume equation gives
\[
V = LWH = (6 + 2W)(W)(63 - W - 6 - 2W) = 342W + 96W^2 - 6W^3
\]
Taking the derivative, setting it equal to zero and solving for \( W \) gives \( W \approx 12.22 \), \( L \approx 30.44 \), and \( H \approx 20.34 \).

8. Installing a power line from \( A \) to \( B \) costs \$2 million per mile. Installing a power line from \( B \) to \( C \) costs \$1 million per mile. If point \( A \) is at \((0, 4)\), point \( B \) is at \((x, 0)\) and point \( C \) is at \((13, 0)\), use calculus to determine the value(s) of \( x \) so that the total cost is minimized.

**Solution:**

The cost, in millions of dollars, is given by
\[
C = 2AB + 1BC = 2\sqrt{16 + x^2} + 13 - x
\]
Taking the derivative and solving gives
\[
\frac{dc}{dx} = \frac{2x}{\sqrt{16 + x^2}} - 1 = 0
\]
\[
\sqrt{16 + x^2} = 2x
\]
\[
3x^2 = 16
\]
\[
x = \frac{4}{\sqrt{3}}
\]

9. A cone is constructed by cutting a sector of angle \( \theta \) from a circular sheet of metal with radius 20 cm. The cut sheet is then folded in a cone. Find a formula for the volume of the cone that involves only the variable \( \theta \) and does not involve \( R \) or \( H \). You do not need to simplify the formula. Note that the volume of a cone is given by \( V = \frac{1}{3}\pi R^2H \).

**Solution:** This problem was done in class. From your notes, you can find
\[
R^2 + H^2 = 400 \quad \text{and} \quad 2\pi R = 40\pi - 20\theta
\]
Substituting these values into the volume equation gives
\[
V = \frac{1}{3}\pi R^2H = \frac{1}{3}\pi R^2\sqrt{400 - R^2} = \frac{1}{3}\pi \left(\frac{20\pi - 10\theta}{\pi}\right)^2 \sqrt{400 - \left(\frac{20\pi - 10\theta}{\pi}\right)^2}
\]