MATH 101A
Chap 4
READ: You must show work to receive credit.

1. Find the critical point(s) of the function \( f(x) = x\sqrt{4-x^2} \).

2. Find the absolute maximum and minimum in the interval \([0, \pi]\) for \( f(x) = 5\cos^2 x \). Answer in point form, \((x, y)\) with exact values.

3. Find an interval(s) where the graph of \( f(x) = x^4 - 6x^2 + 8x + 8 \) is concave up.

4. Suppose that \( f \) is a continuous function that satisfies the following conditions. Sketch the graph of \( y = f(x) \)
   a) \( f(0) = 0, f(2) = 3, \) and \( f(4) = 0 \)
   b) \( f'(0) \) does not exist, \( f'(2) = 0 \)
   c) \( f''(x) < 0 \) for \( x < 0 \) and \( x > 2 \); \( f''(x) > 0 \) for \( 0 < x < 2 \)
   d) \( f''(x) < 0 \) for all \( x, x \neq 0 \)

5. The following figure shows the graphs of \( f, f', \) and \( f'' \). Decide which curve is which.

![Graphs of f, f', and f'']

6. Find the linear approximation of \( f(x) = \ln(2 + x) \) at \( a = -1 \). Use the linear approximation to estimate \( f(-0.5) \).

7. Find the limit.
   \[ \lim_{x \to 1} \frac{\ln x}{\tan(\pi x)} \]

8. Find the limit.
   \[ \lim_{x \to \infty} \left( 1 + \frac{1}{x} \right)^{2x} \]
9. State why the Mean Value Theorem can be applied to \( f(x) = x - \sin x \) on the interval \([-\pi, \pi]\). Then find all numbers in the interval that satisfy the MVT.

10. A right circular cone is inscribed in a hemisphere of radius 10 ft with the apex of the cone and center of the base of the hemisphere being the same point. Find the height of the cone whose volume is the maximum. \( V = \frac{1}{3} \pi r^2 h \)

11. A woman is at point A on the ocean, 4 miles from the nearest point C on a straight shoreline: that point is 6 miles from point B. She wants to reach B in the minimum time. She is going to row a boat to point D and walk to point B. If she can row the boat at 2 miles per hour and walk at 3 miles per hour, at which point on shore, does she need to land so that the total time is the minimum? Hint: Let \( x \) be the distance between C and D, at which she lands and find \( x \). You may assume the minimum time is yielded at the critical point.