1. Given the velocity $v(t)$ function shown in the drawing, sketch the graph of the position function $x(t)$. Assume that $x(0) = 0$, and label the points on the graph at $t = 2$ and $t = 4$.

2. Draw the solution curves through $(-1.5, 0.5)$ and $(-0.5, 0.5)$.

3. Solve the initial value problem $y' = 3x^2(y^2 + 1)$, $y(0) = 1$. The solution should be written in the form $y = f(x)$.

**Solution:** Use the method of separation of variables to get $y(x) = \tan(\frac{x^3}{4} + \frac{\pi}{4})$.

4. Solve the initial value problem $y' + y \tan x = \cos x$, $y(\pi) = 0$.

**Solution:** Use the method of first order linear equations to get $y(x) = (x - \pi) \cos x$.

5. A tank with capacity 1000 gallons initially contains 200 gallons of solution with 30 pounds of dissolved acid. An acid solution containing 0.5 pounds of acid per gallon flows into the tank at 6 gallons per minute and the well mixed solution flows out at 6 gallons per minute. Set up and solve the initial value problem.
Solution: \[ \frac{dA}{dt} = 3 - \frac{6A}{200}; \quad A(0) = 30 \rightarrow A(t) = 100(1 - 0.7e^{-0.03t}) \]

6. A tank with capacity 1000 gallons initially contains 200 gallons of solution with 30 pounds of dissolved acid. An acid solution containing 0.5 pounds of acid per gallon flows into the tank at 6 gallons per minute and the well mixed solution flows out at 8 gallons per minute. Set up the initial value problem including the time restriction. You do not need to solve the initial value problem.

Solution: \[ \frac{dA}{dt} = 3 - \frac{8A}{200 - 2t}; \quad 0 \leq t \leq 100, \quad A(0) = 30 \]

7. Upon the birth of their first child, a couple deposited $5000 in an account that pays 3% interest compounded continuously. The interest is allowed to accumulate in the account. Determine an expression for the amount of money in the account after 18 years.

Solution: See Section 1.4, problem 37.

8. Sketch the solution curves to \( x'(t) = (x - 1)(x - 3)(x - 5) \). Label the equilibrium solutions as stable, unstable, or semi-stable.

Solution: The solution should indicate that \( x = 1 \) is unstable, \( x = 3 \) is stable, and \( x = 5 \) is unstable.

9. A body moves horizontally through a medium whose resistance is proportional to the square of velocity \( v \), so that \( \frac{dv}{dt} = -kv^2 \). Assuming that the initial velocity is given by \( v(0) = v_0 \) and the initial position is given by \( x(0) = x_0 \), find the position function \( x(t) \).

Solution: See Section 2.3, problem 4

10. At time \( t = 0 \) the bottom plug is removed from a tank of water. The tank is in the shape of a cone, with the point at the bottom. The tank is 16 feet high and the tank is initially full of water. The radius of the top of the tank is 32 feet. After 1 hour, the water in the tank is 9 feet deep. Find an expression for the area of the hole by using Torricelli’s law

\[ a \sqrt{2gy} = -A(y) \frac{dy}{dt} \] where \[ \begin{cases} \text{a = area of the hole} \\ \text{y = height of fluid in the tank} \\ \text{A(y) = area of the cross section of the tank} \end{cases} \]

Solution: See Section 1.4, problem 56

11. Suppose that a projectile is fired straight upward from the surface of the earth with initial velocity \( v_0 < \sqrt{\frac{2GM}{R}} \). It’s height \( y(t) \) above the surface satisfies the initial value problem

\[ \frac{d^2y}{dt^2} = -\frac{GM}{(y + R)^2}; \quad y(0) = 0, \quad y'(0) = v_0 \]

Substitute \( \frac{dv}{dt} = \frac{d}{dy} \) and determine an expression for velocity in terms \( v^2 = \text{your answer} \).
Solution:
\[
\frac{d^2 y}{dt^2} = v \frac{dv}{dy} = - \frac{GM}{(y+R)^2}
\]
\[
\int v \, dv = - \int \frac{GM}{(y+R)^2} \, dy
\]
\[
\frac{v^2}{2} = \frac{GM}{y + R} + C
\]
\[
v^2 = \frac{2GM}{y + R} + C \quad \text{using the initial condition } y'(0) = v_0 \text{ gives}
\]
\[
v_0^2 = \frac{2GM}{R} + C \quad \rightarrow \quad C = v_0^2 - \frac{2GM}{R}
\]
\[
v^2 = \frac{2GM}{y + R} + v_0^2 - \frac{2GM}{R}
\]