Math 166 Ch. 5 Test

(1) Sketch the feasible region given by

\[ \begin{align*}
  x + y &\leq 16 \\
  5x + y &\leq 40 \\
  3x + 2y &\leq 12 \\
  3x + 4y &\geq 24
\end{align*} \]

Use this graph to find the values of \( x, y \) and \( z \) that both maximize and minimize \( z \).

(2) Use the Simplex method to solve the following. Be sure to state the values of all the variables at the end of the problem.

Maximize \( P = 100x_1 + 300x_2 + 200x_3 \)

subject to:

\[ \begin{align*}
  x_1 + x_2 + x_3 &\leq 100 \\
  40x_1 + 20x_2 + 30x_3 &\leq 3200 \\
  x_1 + 2x_2 + x_3 &\leq 160 \\
  x_1, x_2, x_3 &\geq 0
\end{align*} \]

(3) A feasible region is bounded by the constraints

\[ \begin{align*}
  5x + 2y &\leq 40 \\
  5x + 3y &\leq 30 \\
  x &\geq 0, \ y &\geq 0
\end{align*} \]

A profit function is given by \( P = x + ay \).

(a) For what value(s) of \( a \) will the maximum point occur at the far lower right corner of the feasible region?

(b) For what value(s) of \( a \) will the maximum point occur along the line \( 5x + 2y = 40 \)?

(c) For what value(s) of \( a \) will the maximum point occur at the intersection of the lines \( 5x + 2y = 40 \) and \( x + 3y = 30 \)?

(4) Set up the first Simplex tableau for the following. Be sure to define your variables.

A candy company makes three types of candy, solid-center, fruit-filled, and cream-filled, and packages these candies in two different assortments. A box of assortment I contains 5 solid-centers, 3 fruit-filled and 6 cream filled and sells for $9.40. A box of assortment II contains 8 solid-centers, 7 fruit-filled and 12 cream filled and sells for $21.00. The manufacturing costs per piece of candy are $0.20 for solid-centers, $0.30 for fruit-filled, and $0.40 for cream-filled. The company can manufacture 3000 solid-center, 4000 fruit-filled, and 5000 cream-filled candies weekly. How many boxes of each type should the company produce each week in order to maximize their profit?

State in words the meaning of the first slack variable.
Set up the first objective function and the constraints for the following. Be sure to define your variables.

(5) A department store chain has up to $20,000 to spend on television advertising. All ads will be placed with one television station, where a 30 second ad costs $1,000 on daytime TV and is viewed by 14,000 people, $2,000 on prime-time TV and is viewed by 24,000 people, and $1,500 on late-night TV and is viewed by 18,000 people. The television station will not accept a total of more than 15 ads in all three time periods. How many ads should be placed in each time period in order to maximize the number of people who see the ads?

(6) A company blends long grain rice and wild rice to produce two brands of rice mixes, brand A and brand B. Brand A must contain at least 10% wild rice and brand B must contain at least 5% wild rice. Long grain rice costs the company $0.70 per pound and wild rice costs the company $3.40 per pound. The company sells brand A for $1.50 per pound and brand B for $1.20 per pound. The company has 8000 pounds of long grain rice and 500 pounds of wild rice. How should the company use the available rice to maximize their profit?

(7) A dietitian in a hospital is to arrange a special diet using three types of food L, M, and N. Each ounce of food L contains 20 units of calcium, 10 units of iron, 10 units of vitamin A, and 20 units of cholesterol. Each ounce of food M contains 10 units of calcium, 10 units of iron, 20 units of vitamin A, and 24 units of cholesterol. Each ounce of food N contains 10 units of calcium, 10 units of iron, 10 units of vitamin A, and 18 units of cholesterol. If the minimum daily requirements are 30 units of calcium, 200 units of iron, and 240 units of vitamin A, how many ounces of each food should be used to meet the minimum requirements and at the same time, minimize the cholesterol intake?

(8) A political scientist has received a grant to fund a research project involving voting trends. The budget of the grant includes $3200 for conducting door-to-door interviews the day before an election. Undergraduate students, graduate students, and faculty members will be hired to conduct the interviews. Each undergraduate student will be conduct 18 interviews and be paid $100. Each graduate student will conduct 25 interviews and be paid $150. Each faculty member will conduct 30 interviews and be paid $200. Due to limited transportation facilities, no more that 20 interviewers can be hired. One of the conditions of the grant is that at least 50% of the interviewers must be undergraduate students. How many undergraduate students, graduate students, and faculty members should be hired in order to maximize the number of interviews that will be conducted?

(9) A farmer grows three crops, corn oats, and soybeans, which he mixes together to feed his cows and pigs. At least 40% of the feed mix for the cows must be corn. The feed mix for the pigs must contain at least twice as much soybeans as corn. He has harvested 1000 bushels of corns, 500 bushel of oats, and 1000 bushels of soybeans. He needs 1000 bushels of each feed mix for his livestock. The unused corn, oats and soybeans can sold for $4, $3.50, and $3.25 a bushel, respectively (thus, these amounts also represent the cost of the crops used to feed the livestock.) How many bushels of each crop should be used in each feed mix in order to produce sufficient food for the livestock at minimal cost?