1. Perform one pivot operation on the following simplex tableau. (10 pts)

\[
\begin{array}{ccccccc}
\text{x}_1 & \text{x}_2 & \text{x}_3 & \text{s}_1 & \text{s}_2 & \text{P} \\
\text{s}_1 & 6 & 3 & 3 & 1 & 0 & 0 & 24 \\
\text{s}_2 & 4 & -1 & 2 & 0 & 1 & 0 & 32 \\
\text{P} & -4 & 2 & -6 & 0 & 0 & 1 & 0 \\
\end{array}
\]

2. Minimize \( C = 5\text{x}_1 + 8\text{x}_2 \)
Subject to
\[
\begin{align*}
\text{x}_1 + \text{x}_2 & \geq 48 \\
\text{x}_1 + 4\text{x}_2 & \geq 96 \\
3\text{x}_1 + 2\text{x}_2 & \geq 30 \\
\text{x}_1, \text{x}_2 & \geq 0
\end{align*}
\]

The minimization problem above was solved using the dual method, giving the final tableau shown below.

\[
\begin{array}{ccccccc}
\text{y}_1 & \text{y}_2 & \text{y}_3 & \text{x}_1 & \text{x}_2 & \text{P} \\
1 & 0 & 10 & 4 & -1 & 0 & 4 \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
0 & 1 & -1 & -1 & 1 & 0 & 1 \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
0 & 0 & 98 & 32 & 16 & 1 & 288 \\
\end{array}
\]

What is the solution to the minimization problem? (6 pts)
3. Maximize \( P = 4x_1 + 8x_2 + 2x_3 \)
Subject to
\[
\begin{align*}
2x_1 + 2x_2 + 3x_3 & \leq 12 \\
2x_1 - 2x_2 + x_3 & = 4 \\
x_1, x_2, x_3 & \geq 0
\end{align*}
\]

The maximization problem above was solved using the Big M method, giving the final tableau shown below.

\[
\begin{array}{cccccc|c}
 x_1 & x_2 & x_3 & s_1 & a_1 & P \\
 \hline
 0 & 1 & 1 & 1 & -1 & 0 & 2 \\
 & 0 & 0 & 0 & 0 & 0 & 0 \\
 1 & 0 & 1 & 1 & 1 & 0 & 4 \\
 & 0 & 0 & 6 & 3 & M-1 & 1 & 32 \\
\end{array}
\]

What is the solution to the maximization problem? (6 pts)

4. Minimize \( C = 6x_1 + 10x_2 + 3x_3 \)
Subject to
\[
\begin{align*}
2x_1 + 5x_2 + 4x_3 & \geq 12 \\
7x_1 + x_2 + 8x_3 & \geq 25 \\
x_1, x_2, x_3 & \geq 0
\end{align*}
\]

a) Find the dual problem to the minimization problem above. (10 pts)

b) Write the initial simplex tableau for the dual problem. (9 pts)

5. Maximize \( P = 4x_1 + 6x_2 \)
Subject to
\[
\begin{align*}
x_1 + 3x_2 & \leq 12 \\
2x_1 + x_2 & \geq 10 \\
x_1, x_2 & \geq 0
\end{align*}
\]

Find the initial simplex tableau and identify the first pivot element. (11 pts)

6. Solve the linear programming problem. (15 pts)

Maximize \( P = 3x_1 + 8x_2 \)
Subject to
\[
\begin{align*}
x_1 + 2x_2 & \leq 80 \\
2x_1 + x_2 & \leq 64 \\
x_1, x_2 & \geq 0
\end{align*}
\]
7. (9 pts) Set up the following linear programming problem. Do NOT solve the problem.

A dietitian in a hospital is to arrange a special diet using two foods, M and N. Each ounce of food M contains 12 units of calcium, 8 units of iron, 15 units of vitamin A, and 20 units of cholesterol. Each ounce of food N contains 10 units of calcium, 14 units of iron, 9 units of vitamin A, and 16 units of cholesterol. If the minimum daily requirements are 300 units of calcium, 200 units of iron, and 240 units of vitamin A, how many ounces of each food should be used to minimize the cholesterol intake?

8. \( n(A) = 25, \quad n(B) = 40, \quad n(A \cup B) = 52, \quad n(U) = 70 \)

Use the information above to fill in the following table. (10 pts)

<table>
<thead>
<tr>
<th>A</th>
<th>A′</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B′</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

9. (6 pts each) Find each of the following without using any special calculator functions.
   a) \( C_{12,8} \)
   a) \( P_{25,2} \)

10. Rob has a deck of 40 cards, 30 of which are black and 10 of which are red. How many 5-card hands will have 2 black cards and 3 red cards? (10 pts)