Math 101B Ch. 10 Test
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1. (10 points) Find the equation of the line in parametric form that passes through the points $(8, 2)$ and $(-3, 6)$.

Solution:
$$x = 8 - 11t, \quad y = 2 + 4t$$

2. (10 points) Find the equation of the circle in parametric form that has radius $r = 5$, center $(2, 3)$ and completes one full revolution on the period $t \in [0, 4]$. The answer should be written in terms of sine and cosine.

Solution:
$$x = 2 + 5\cos\frac{\pi t}{2}, \quad y = 3 + 5\sin\frac{\pi t}{2}$$

3. (10 points) Find the parametric equation of the tangent line to the parabola
$$x = t, y = 10 + 4t - t^2$$
at the vertex of the parabola.

Solution: Since the vertex occurs when $y' = 0$, the point of tangency is at $(2, 14)$. Since the slope at this point is zero, the equation of the tangent line is given by
$$x = t, y = 14$$

4. (10 points) Find the value(s) of t where the curve $x = \sin 4t$, $y = \sin 2t$ has a vertical tangent line.

Solution: The vertical tangent line will occur when $\frac{dx}{dt} = 4\cos 4t = 0$. This gives $t = \frac{\pi}{8} + \frac{\pi}{4}k$.

5. (10 points) Find the slopes of the two tangents lines to the polar function $r = 2 + 3\cos\theta$ when the curve crosses the origin.

Solution: The curve crosses the origin at $r = 0$. Solving gives $\theta = 2.3005$ and $\theta = 3.9827$. Thus the slope is $\tan\theta = \pm 1.1180$.

6. (10 points) Find all solutions to the equation $\cos 2t \sin t + 3\sin 2t \cos t = 0$.

Solution:
$$\begin{aligned} \cos 2t \sin t + 3\sin 2t \cos t &= 0 \\ (\cos^2 t - \sin^2 t)\sin t + 3(2\cos t \sin t)\cos t &= 0 \\ 7\cos^2 t \sin t - \sin^3 t &= 0 \\ \sin t(8\cos^2 t - 1) &= 0 \end{aligned}$$

Therefore, $\sin t = 0 \longrightarrow t = \pi k$ and
$$\cos t = \pm\frac{1}{2\sqrt{2}} \longrightarrow t = 1.2094 + 2\pi k,\ t = 1.9322 + 2\pi k,\ t = 4.3510 + 2\pi k,\ t = 5.0738 + 2\pi k$$

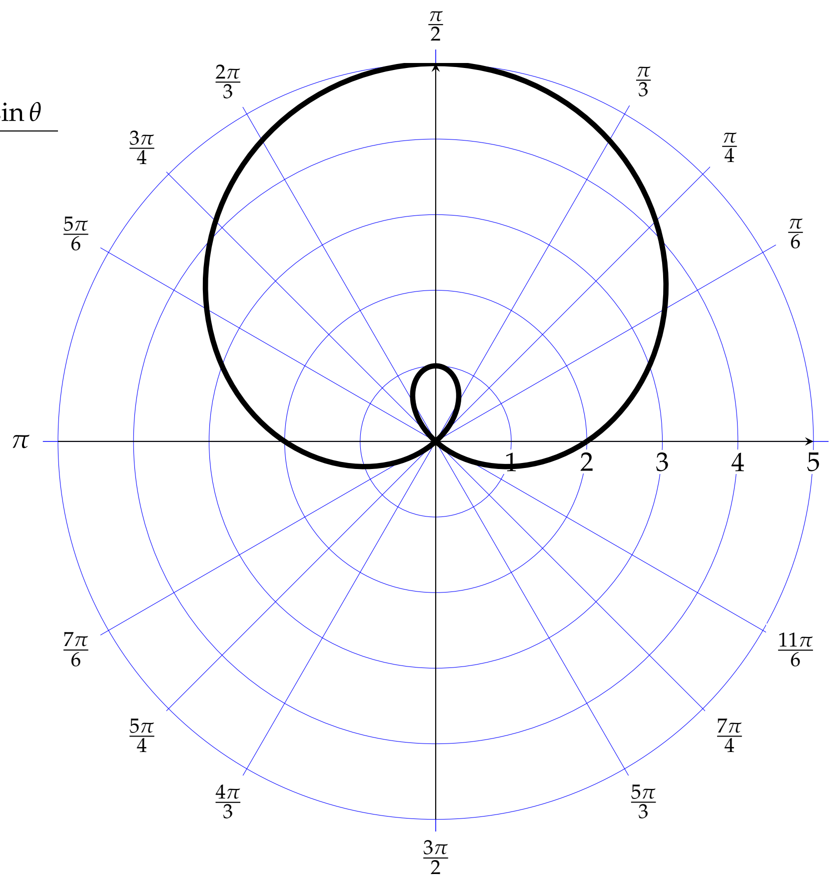
7. (10 points) Convert the parametric equations $x = t,\ y = t - 16t^2$ into a polar function $r = f(\theta)$.

Solution: Converting the equation to rectangular gives $y = x - 16x^2$. Converting this to polar gives $r\sin\theta = r\cos\theta - 16r^2\cos^2\theta$. Solving for r gives

$$r = \frac{\cos\theta - \sin\theta}{16\cos^2\theta}$$

8. (10 points) Sketch the graph of the polar equation $r = 2 + 3\sin\theta$ on the interval $t \in [\pi, 2\pi]$. Be sure to complete the following table.

θ	$r = 2 + 3\sin\theta$
π	
$\frac{7\pi}{6}$	
$\frac{5\pi}{4}$	
$\frac{4\pi}{3}$	
$\frac{3\pi}{2}$	
$\frac{5\pi}{3}$	
$\frac{7\pi}{4}$	
$\frac{11\pi}{6}$	
2π	



9. Set up the integral(s) to find the indicated areas. You do not need to solve the integration.

(a) (10 points) The region inside the curve $r = \sin 2\theta$.

Solution:

$$8 \cdot \frac{1}{2} \int_0^{\pi/4} \sin^2 2\theta \, d\theta$$

(b) (10 points) The region inside the curve $r = \sin\theta$.

Solution:

$$\frac{1}{2} \int_0^{\pi} \sin^2 \theta \, d\theta$$

(c) (10 points) The region in the first quadrant inside the curve $r = 5\sin 3\theta$ and outside $r = 2$.

Solution: Setting the functions equal to each and solving for θ gives $\theta = 0.1372$ and $\theta = 0.9100$. Using these to set up the integral gives

$$\frac{1}{2} \int_{0.1372}^{0.9100} \left(25 \sin^2 3\theta - 4\right) \, d\theta$$