MATH 101B

Chapter 9

1. Let $f(x)=\frac{2}{x^2}$.

a) Find the fourth Taylor polynomial for $f(x)$ centered at $x = 1$.

b) Using part (a), approximate $\frac{2}{(1.1)^2}$

2. Use the first four terms of the Taylor series for the function $f(x)=\sqrt{x}$ centered at $a=4$ to approximate the value of $\sqrt{5}$.

3. Let f be the function $f(x)=\sum_{k=0}^{\infty}\frac{(-1)^k(x-4)^k}{k+1}$.

a) Find the radius of convergence of $f(x)$.

b) Find the interval of convergence. Make sure to check the endpoints.

b) Find the power series for $\int f(x)dx$.

4. Find a function, $f(x)$ for the following power series. Simplify the function.

$$\sum_{k=0}^{\infty}\frac{(-1)^k(x-1)^k}{5^{2k}}$$

5. Find the power series centered at $a = 0$ for the following function. Answer in sigma notation.

$h(x)=\frac{x}{(1+x^2)^2}$ using $f(x)=\frac{1}{1+x^2}$

6. Using the power series for $\cos x$, approximate the value of $\int_0^1 \cos\sqrt{x}\,dx$ with an error of less than 0.0001.

7. a) Find a power series for the solution to the following differential equation. Give the sigma notation.

$y'(t)-y(t)=0,\ y(0)=3$ Hint: $y(t)=\sum_{k=0}^{\infty}c_k t^k$ and $c_k=\frac{y^{(k)}(0)}{k!}$

b) Identify the function represented by the power function in (a).